CCSC Mathematics

Maths Crucial Knowledge Year 10-11 [Higher]

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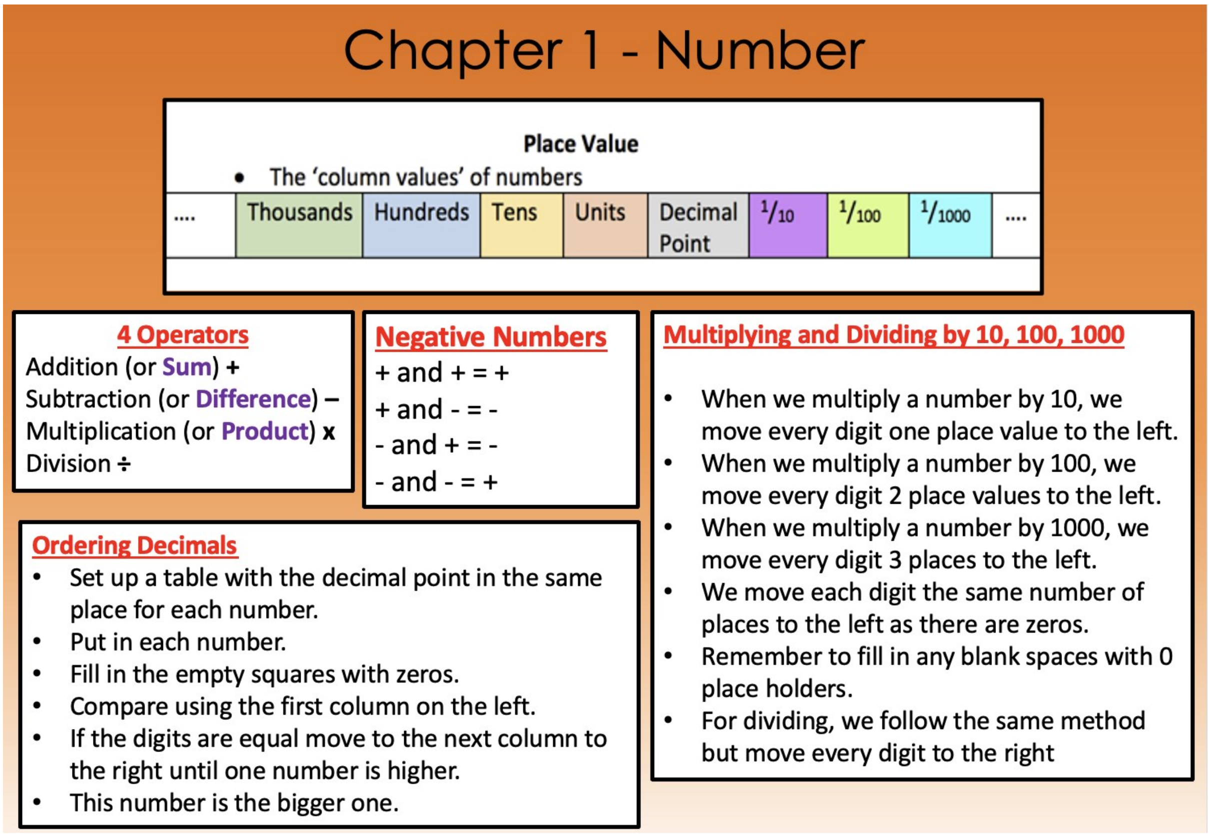
Number



Chapter 1 - Number

Place Value

- The 'column values' of numbers

....	Thousands	Hundreds	Tens	Units	Decimal Point	$^1/_{10}$	$^1/_{100}$	$^1/_{1000}$	....

4 Operators

Addition (or **Sum**) +
Subtraction (or **Difference**) –
Multiplication (or **Product**) x
Division ÷

Negative Numbers

+ and + = +
+ and - = -
- and + = -
- and - = +

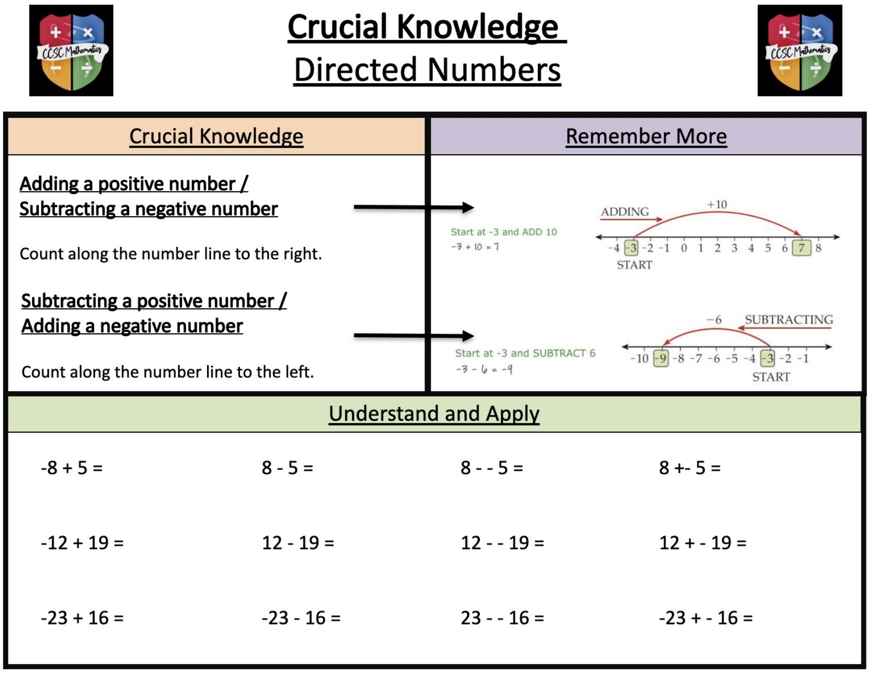
Multiplying and Dividing by 10, 100, 1000

- When we multiply a number by 10, we move every digit one place value to the left.
- When we multiply a number by 100, we move every digit 2 place values to the left.
- When we multiply a number by 1000, we move every digit 3 places to the left.
- We move each digit the same number of places to the left as there are zeros.
- Remember to fill in any blank spaces with 0 place holders.
- For dividing, we follow the same method but move every digit to the right

Ordering Decimals

- Set up a table with the decimal point in the same place for each number.
- Put in each number.
- Fill in the empty squares with zeros.
- Compare using the first column on the left.
- If the digits are equal move to the next column to the right until one number is higher.
- This number is the bigger one.

Know More, Remember More, Understand and Apply



Crucial Knowledge
Directed Numbers

Crucial Knowledge

Adding a positive number / Subtracting a negative number

Count along the number line to the right.

Subtracting a positive number / Adding a negative number

Count along the number line to the left.

Remember More

Start at -3 and ADD 10
-3 + 10 = 7

Start at -3 and SUBTRACT 6
-3 - 6 = -9

Understand and Apply

-8 + 5 =	8 - 5 =	8 - - 5 =	8 +- 5 =
-12 + 19 =	12 - 19 =	12 - - 19 =	12 + - 19 =
-23 + 16 =	-23 - 16 =	23 - - 16 =	-23 + - 16 =

Crucial Knowledge
Highest Common Factor [Listing]

Crucial Knowledge	Remember More
The Highest Common Factor is the biggest number that goes into two or more given numbers.	Highest Common Factor of 24 and 36:
Write down all the factors of each number. →	**24** – 1, 2, 3, 4, 6, 8, 12, 24 **36** – 1, 2, 3, 4, 6, 9, 12, 18, 36
Identify the biggest number they both have in the list. →	They both have 1, 2, 3, 4, 6 and 12. 12 is the biggest they both have. It is the highest common factor.

Understand and Apply

Using this listing method, find the highest common factor of...

a) 16 and 24

b) 27 and 45

c) 16 and 32

Crucial Knowledge
Lowest Common Multiple [Listing]

Crucial Knowledge	Remember More
The Lowest Common Multiple is the smallest number that is in the times tables of two or more numbers.	Lowest Common Multiple of 6 and 8:
Write down part of the times tables for each number. →	**6** – 6, 12, 18, **24**, 30, 36, 42, 48, 54 **8** – 8, 16, **24**, 32, 40, 48, 56
Identify the smallest number they both have. →	They both have 24 and 48 in their times tables. 24 is the smallest, so that is the lowest common multiple.

Understand and Apply

Using this listing method, find the lowest common multiple of...

a) 4 and 7

b) 3 and 9

c) 11 and 2

Crucial Knowledge

Rounding

Crucial Knowledge

If the digit to the right of the place value you are rounding to is…

- 5 or more: round up
- Less than 5: round down

Remember More

To the nearest hundred…

Th H T U
7384 = 7400

Th H T U
7344 = 7300

Understand and Apply

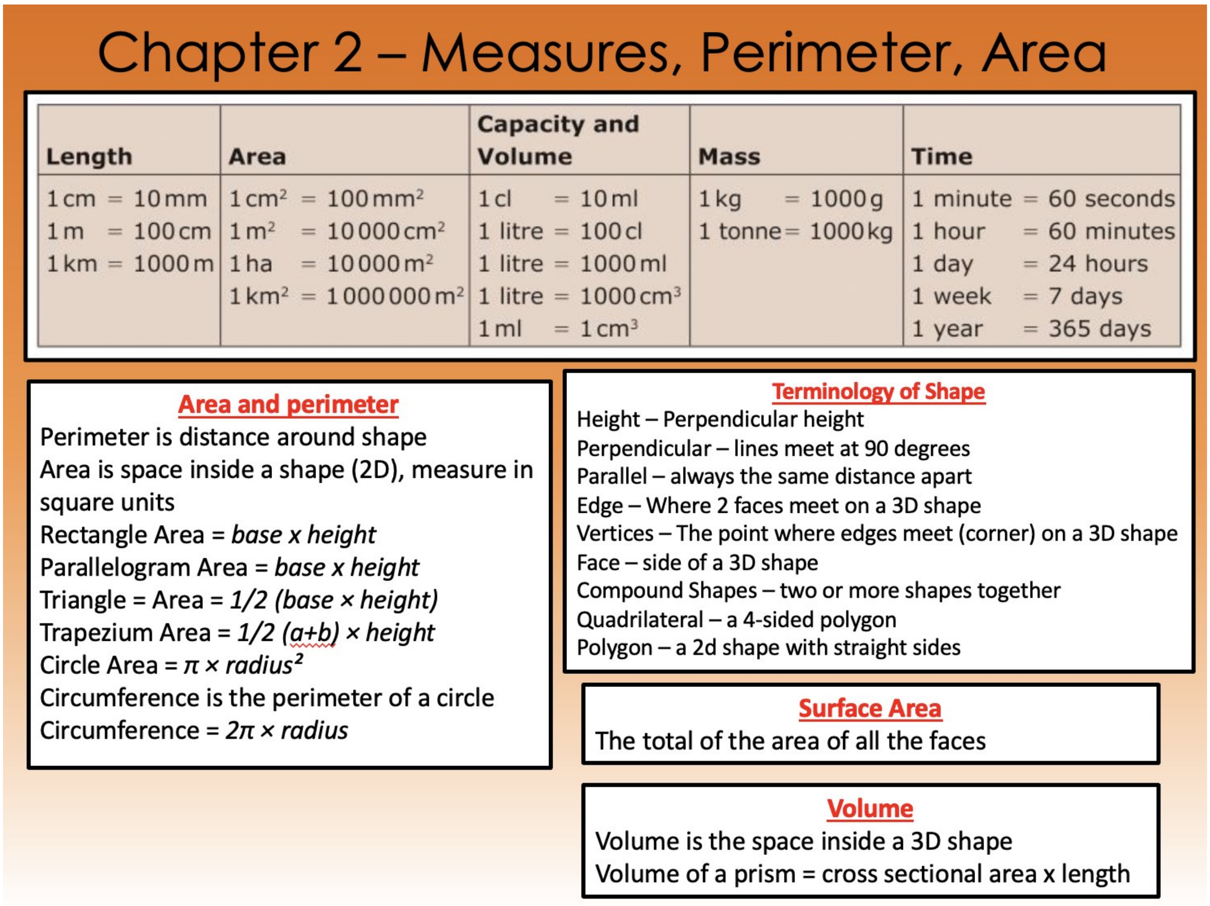
1. Round to the nearest hundred:

a) 7354 b) 882 c) 94

2. Round to the nearest ten:

a) 664 b) 3078 c) 13

Measures, Area, Perimeter



Chapter 2 – Measures, Perimeter, Area

Length	Area	Capacity and Volume	Mass	Time
1 cm = 10 mm	1 cm^2 = 100 mm^2	1 cl = 10 ml	1 kg = 1000 g	1 minute = 60 seconds
1 m = 100 cm	1 m^2 = 10 000 cm^2	1 litre = 100 cl	1 tonne = 1000 kg	1 hour = 60 minutes
1 km = 1000 m	1 ha = 10 000 m^2	1 litre = 1000 ml		1 day = 24 hours
	1 km^2 = 1 000 000 m^2	1 litre = 1000 cm^3		1 week = 7 days
		1 ml = 1 cm^3		1 year = 365 days

Area and perimeter

Perimeter is distance around shape
Area is space inside a shape (2D), measure in square units
Rectangle Area = *base x height*
Parallelogram Area = *base x height*
Triangle = Area = *1/2 (base × height)*
Trapezium Area = *1/2 (a+b) × height*
Circle Area = $\pi \times radius^2$
Circumference is the perimeter of a circle
Circumference = $2\pi \times radius$

Terminology of Shape

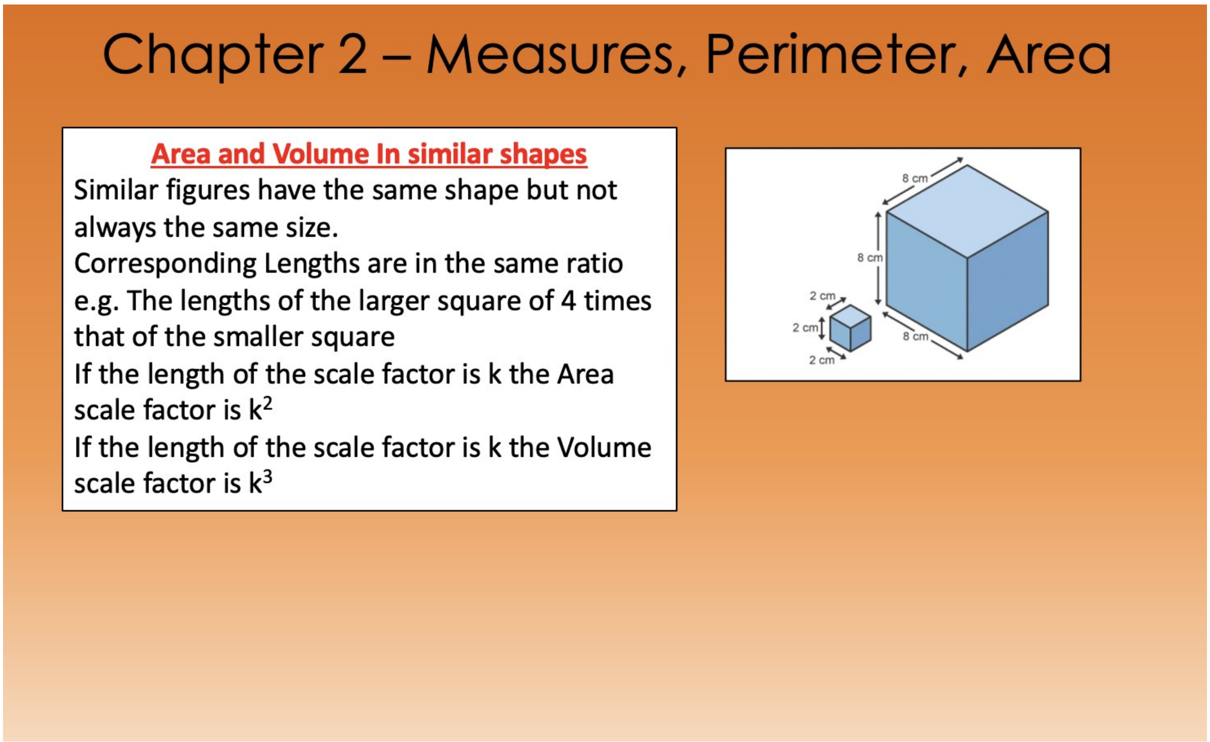
Height – Perpendicular height
Perpendicular – lines meet at 90 degrees
Parallel – always the same distance apart
Edge – Where 2 faces meet on a 3D shape
Vertices – The point where edges meet (corner) on a 3D shape
Face – side of a 3D shape
Compound Shapes – two or more shapes together
Quadrilateral – a 4-sided polygon
Polygon – a 2d shape with straight sides

Surface Area

The total of the area of all the faces

Volume

Volume is the space inside a 3D shape
Volume of a prism = cross sectional area x length

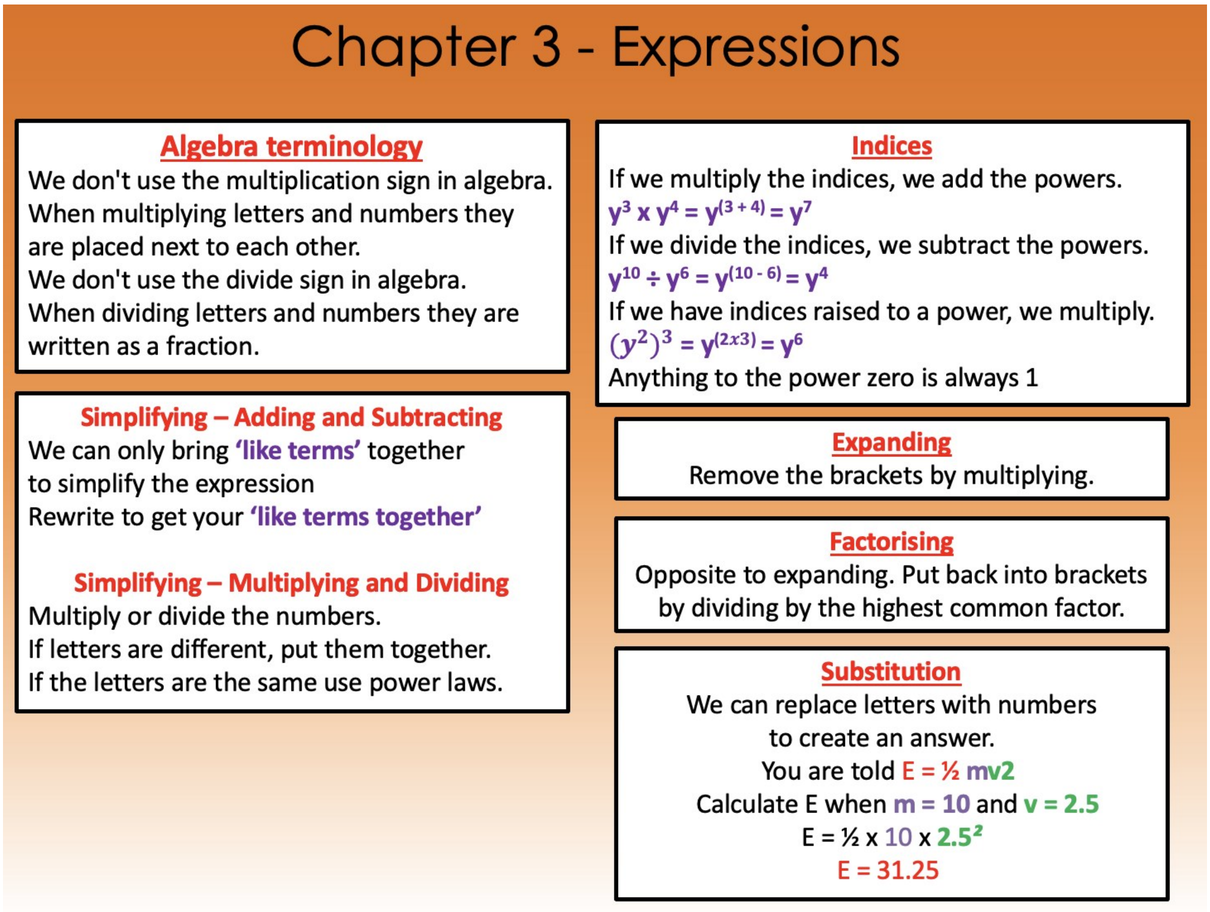


Chapter 2 – Measures, Perimeter, Area

Area and Volume In similar shapes

Similar figures have the same shape but not always the same size.
Corresponding Lengths are in the same ratio e.g. The lengths of the larger square of 4 times that of the smaller square
If the length of the scale factor is k the Area scale factor is k^2
If the length of the scale factor is k the Volume scale factor is k^3

Expressions



Chapter 3 - Expressions

Algebra terminology

We don't use the multiplication sign in algebra.
When multiplying letters and numbers they are placed next to each other.
We don't use the divide sign in algebra.
When dividing letters and numbers they are written as a fraction.

Simplifying – Adding and Subtracting

We can only bring **'like terms'** together to simplify the expression
Rewrite to get your **'like terms together'**

Simplifying – Multiplying and Dividing

Multiply or divide the numbers.
If letters are different, put them together.
If the letters are the same use power laws.

Indices

If we multiply the indices, we add the powers.
$y^3 \text{ x } y^4 = y^{(3+4)} = y^7$
If we divide the indices, we subtract the powers.
$y^{10} \div y^6 = y^{(10-6)} = y^4$
If we have indices raised to a power, we multiply.
$(y^2)^3 = y^{(2x3)} = y^6$
Anything to the power zero is always 1

Expanding

Remove the brackets by multiplying.

Factorising

Opposite to expanding. Put back into brackets by dividing by the highest common factor.

Substitution

We can replace letters with numbers to create an answer.
You are told E = ½ mv2
Calculate E when m = 10 and v = 2.5
E = ½ x 10 x 2.5^2
E = 31.25

Chapter 3 - Expressions

Using Indices in ALgebra

Reciprocal means 'to the power of minus 1' or 'one over'
e.g. The reciprocal of 2 is 2^{-1} or ½

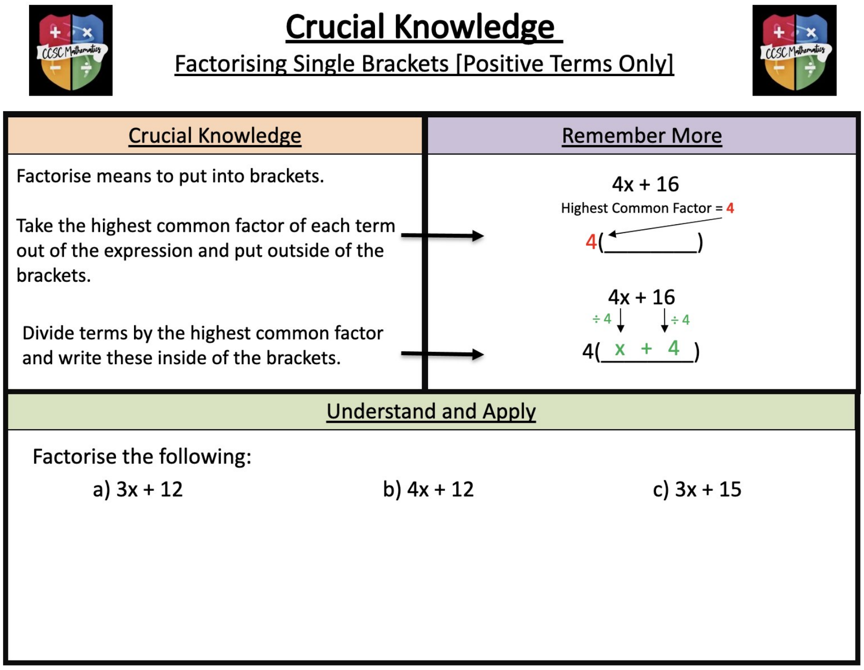
Fractions in Algebra

To simplify Algebraic Fractions you treat them in the same way as normal fractions.

Rearranging Formula

The subject of a formula is the Variable that is on one side on its own. It is expressed in terms of the other variable.
To 'change the subject' of a formula, isolate the new subject , step by step.

Know More, Remember More, Understand and Apply It



Crucial Knowledge

Factorising Single Brackets [Positive Terms Only]

Crucial Knowledge	Remember More
Factorise means to put into brackets. Take the highest common factor of each term out of the expression and put outside of the brackets.	$4x + 16$ Highest Common Factor = 4 $4(\_\_\_\_\_)$
Divide terms by the highest common factor and write these inside of the brackets.	$4x + 16$ $\div 4$ $\div 4$ $4(x + 4)$

Understand and Apply

Factorise the following:

a) $3x + 12$ b) $4x + 12$ c) $3x + 15$

1 - Factorising Single Brackets [Positive Terms Only]

Crucial Knowledge

Factorising Single Brackets [Multiple Letters/Squared Terms]

Crucial Knowledge

Factorise means to put into brackets.

Take the highest common factor of each term out of the expression and put outside of the brackets. Sometimes this highest common factor can be **or** include a letter.

Divide terms by the HCF and write these inside of the brackets.

Remember More

$3xy + 9x$

Highest Common Factor = 3 AND x

$3x(\_\_\_\_\_\_)$

$3xy + 9x$

÷ 3x ÷ 3x

$3x(\underline{\ y + 3\ })$

Understand and Apply

Factorise the following:

a) $xy + 2x$ b) $2xy + 4y$ c) $2x^2 + 6x$

2 - Factorising Single Brackets [Multiple Letters/Squared Terms]

Crucial Knowledge

Factorising Single Brackets [Negative Terms]

Crucial Knowledge

Factorise means to put into brackets.

Take the highest common factor of each term out of the expression and put outside of the brackets.

Divide terms by the HCF and write these inside of the brackets.

A negative divided by a positive is a negative. A negative divided by a negative is a positive.

Remember More

$4x - 16$

Highest Common Factor = 4

$4(\_\_\_\_\_\_)$

$4x - 16$

÷ 4 ÷ 4

$4(\underline{\ x - 4\ })$

Understand and Apply

Factorise the following:

a) $5x + 20$ b) $5x - 20$ c) $-5x - 20$

3 - Factorising Single Brackets [Negative Terms]

Crucial Knowledge
Expanding Single Brackets (Positives)

Crucial Knowledge	Remember More
Remove the brackets by multiplying	
Multiply the term on the outside of the bracket by the first term inside the bracket	3 (x + 4) 3 x X = 3x
Multiply the term on the outside of the bracket by the second term in the bracket	3 (x + 4) 3 x 4 = +12
Bring the two multiplied terms together	3x +12

Understand and Apply

Expand the following brackets:

5 (x + 7)	x (x + 4)	3x (x + 2)
7 (2 + y)	y (3 + y)	2y (3 + 4y)
2 (z + 5)	z (z + 6)	z^2 (z + 6z)

4 - Expanding Single Brackets [Positive Terms]

Crucial Knowledge
Expanding Single Brackets (Negatives)

Crucial Knowledge	Remember More
Remove the brackets by multiplying	
Multiply the term on the outside of the bracket by the first term inside the bracket	3 (x - 4) 3 x -X = 3x
Multiply the term on the outside of the bracket by the second term in the bracket	3 (x - 4) 3 x -4 = -12
Bring the two multiplied terms together	3x -12

Understand and Apply

Expand the following brackets:

5 (x - 7)	x (x - 4)	3x (x - 2)
7 (2 - y)	y (3 - y)	2y (3 - 4y)
2 (z - 5)	z (z - 6)	z^2 (z - 6z)

5 - Expanding Single Brackets [Negative Terms]

Crucial Knowledge

Difference of Two Squares [No Coefficient of x^2]

Crucial Knowledge

Difference of Two Squares is a method of factorising that can be used when the expression is in the form:

$x^2 - c$

where 'c' is a square number

Factorise into two brackets, with one positive square root and one negative square root of c.

Remember More

$$x^2 - 64$$

Square root of 64 is 8...

$$(x \underline{+ 8})(x \underline{- 8})$$

Understand and Apply

Factorise the following:

a) $x^2 - 9$

b) $x^2 - 16$

c) $x^2 - 100$

6 - Factorising Difference of Two Squares [No Coefficient for x squared]

Crucial Knowledge

Difference of Two Squares [Square Coefficient of x^2]

Crucial Knowledge

Can be used when:

$x^2 - c$

where 'c' is a square number

Square root the coefficient of x^2 and make this the coefficient of x in each bracket.

Fill one bracket with the positive square root and one with the negative square root of c.

Remember More

$$9x^2 - 64$$

$$(3x \quad)(3x \quad)$$

Square root of 64 is 8...

$$(3x \underline{+ 8})(3x \underline{- 8})$$

Understand and Apply

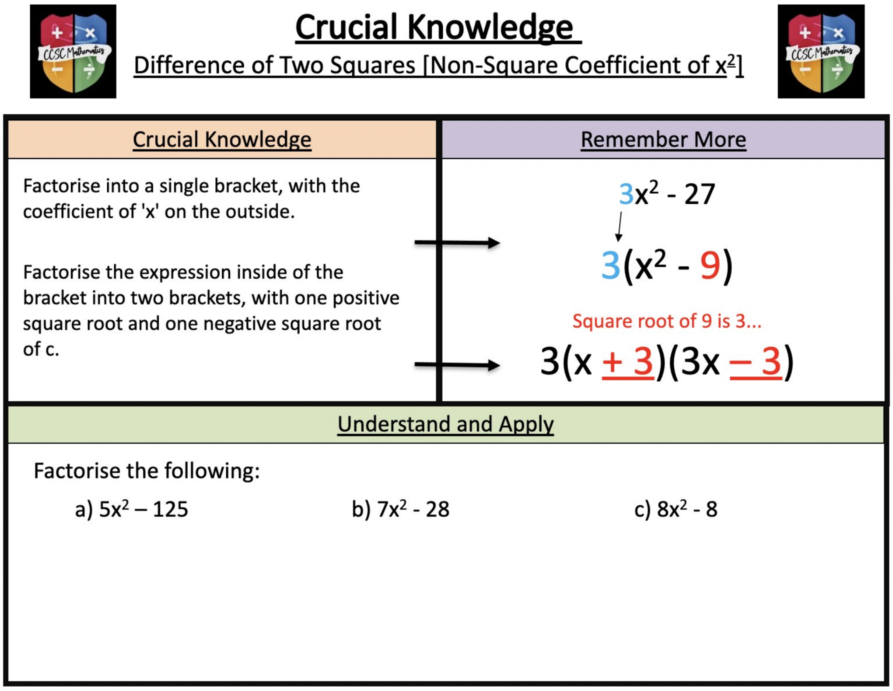
Factorise the following:

a) $4x^2 - 9$

b) $16x^2 - 9$

c) $36x^2 - 9$

7 - Factorising Difference of Two Squares [Square Coefficient for x squared]



Crucial Knowledge

Difference of Two Squares [Non-Square Coefficient of x^2]

Crucial Knowledge	Remember More
Factorise into a single bracket, with the coefficient of 'x' on the outside.	$3x^2 - 27$ $3(x^2 - 9)$
Factorise the expression inside of the bracket into two brackets, with one positive square root and one negative square root of c.	Square root of 9 is 3… $3(x \underline{+ 3})(3x \underline{- 3})$

Understand and Apply

Factorise the following:

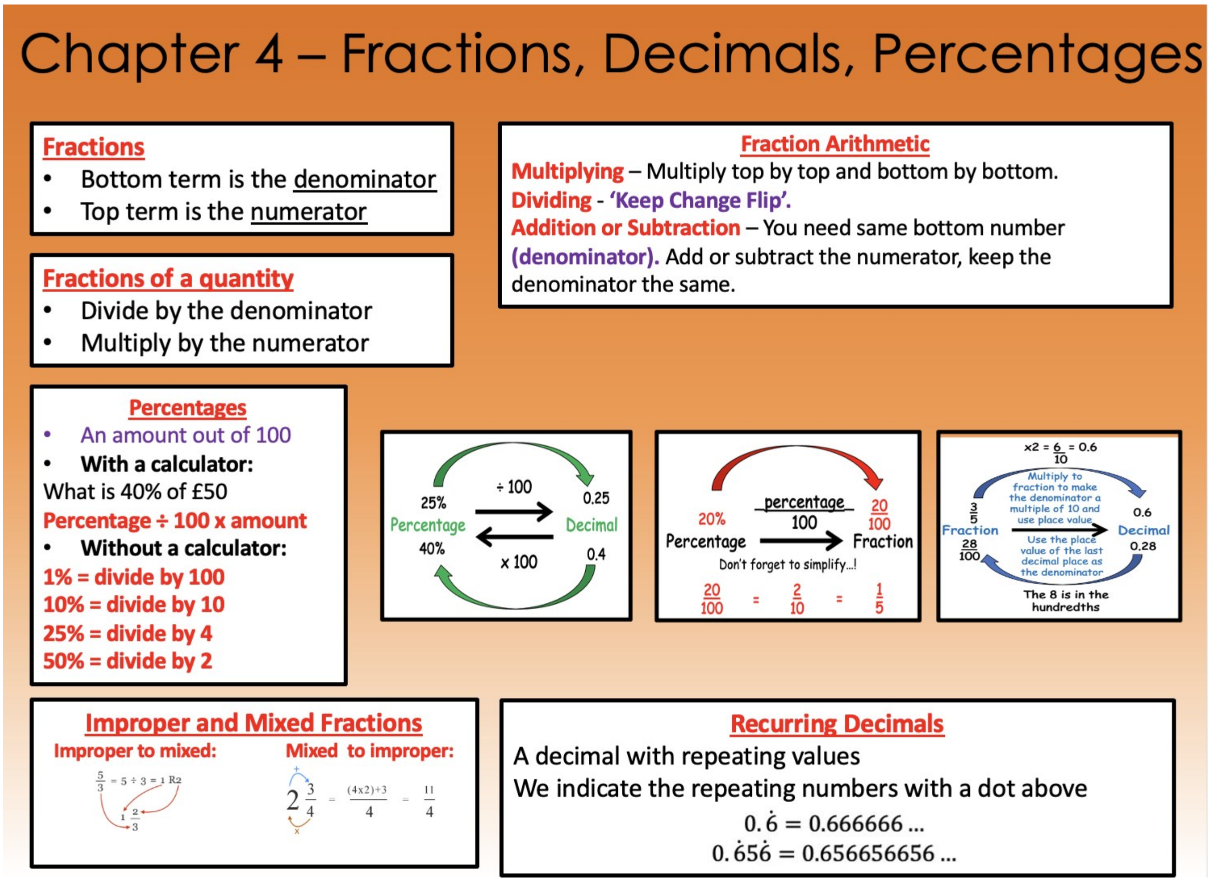
a) $5x^2 – 125$

b) $7x^2 - 28$

c) $8x^2 - 8$

8 - Factorising Difference of Two Squares [Non-quare Coefficient for x squared]

Fractions, Decimals, Percentages



Chapter 4 – Fractions, Decimals, Percentages

Fractions
- Bottom term is the denominator
- Top term is the numerator

Fractions of a quantity
- Divide by the denominator
- Multiply by the numerator

Fraction Arithmetic

Multiplying – Multiply top by top and bottom by bottom.

Dividing - **'Keep Change Flip'.**

Addition or Subtraction – You need same bottom number **(denominator).** Add or subtract the numerator, keep the denominator the same.

Percentages
- An amount out of 100
- **With a calculator:**

What is 40% of £50

Percentage ÷ 100 x amount
- **Without a calculator:**

1% = divide by 100

10% = divide by 10

25% = divide by 4

50% = divide by 2

25% Percentage 40% ÷ 100 → 0.25 Decimal 0.4 ← x 100

20% Percentage → $\frac{\text{percentage}}{100}$ → $\frac{20}{100}$ Fraction

Don't forget to simplify…!

$\frac{20}{100} = \frac{2}{10} = \frac{1}{5}$

x2 = $\frac{6}{10}$ = 0.6

$\frac{3}{5}$ Fraction → Decimal 0.6

Multiply to fraction to make the denominator a multiple of 10 and use place value

$\frac{28}{100}$ ← 0.28

Use the place value of the last decimal place as the denominator

The 8 is in the hundredths

Improper and Mixed Fractions

Improper to mixed:

$\frac{5}{3} = 5 \div 3 = 1\ R2$

$1\frac{2}{3}$

Mixed to improper:

$2\frac{3}{4} = \frac{(4x2)+3}{4} = \frac{11}{4}$

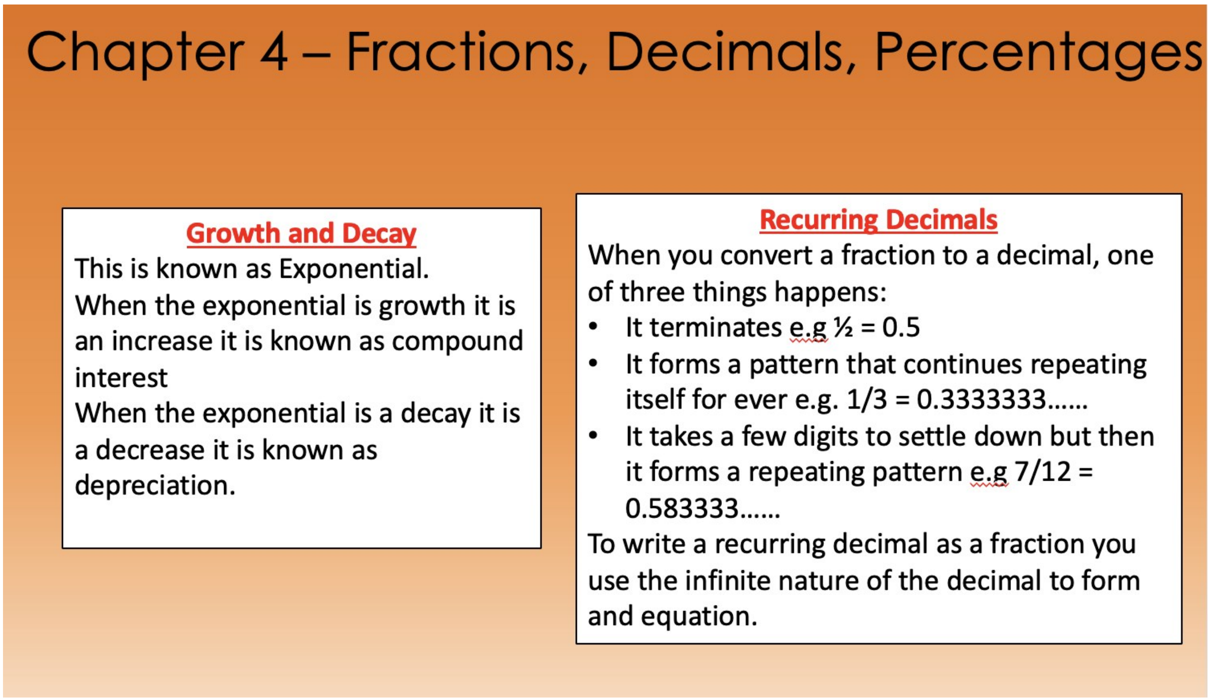
Recurring Decimals

A decimal with repeating values

We indicate the repeating numbers with a dot above

$0.\dot{6} = 0.666666\ldots$

$0.\dot{6}5\dot{6} = 0.656656656\ldots$



Chapter 4 – Fractions, Decimals, Percentages

Growth and Decay

This is known as Exponential.
When the exponential is growth it is an increase it is known as compound interest
When the exponential is a decay it is a decrease it is known as depreciation.

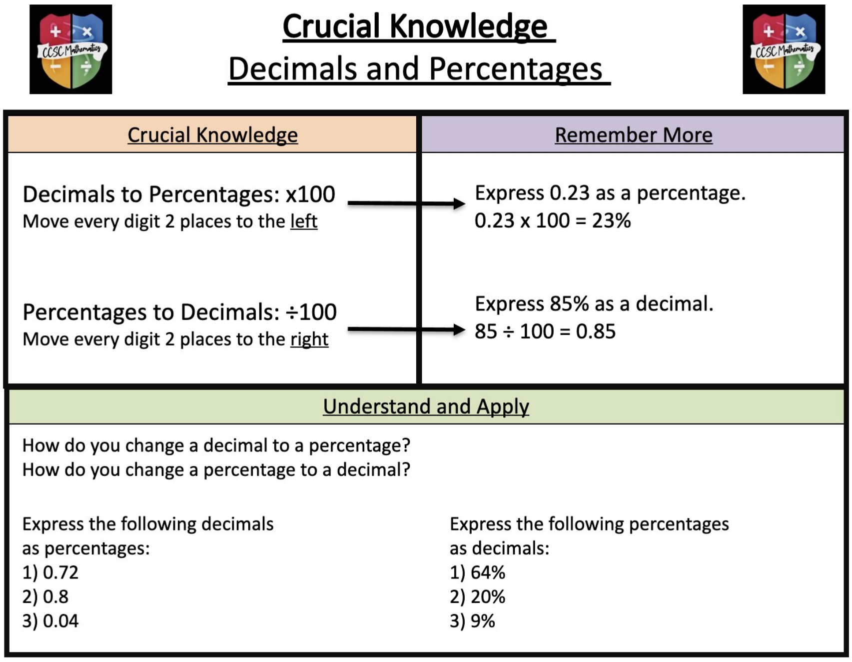
Recurring Decimals

When you convert a fraction to a decimal, one of three things happens:

- It terminates e.g ½ = 0.5
- It forms a pattern that continues repeating itself for ever e.g. 1/3 = 0.3333333……
- It takes a few digits to settle down but then it forms a repeating pattern e.g 7/12 = 0.583333……

To write a recurring decimal as a fraction you use the infinite nature of the decimal to form and equation.

Know More, Remember More, Understand and Apply



Crucial Knowledge
Decimals and Percentages

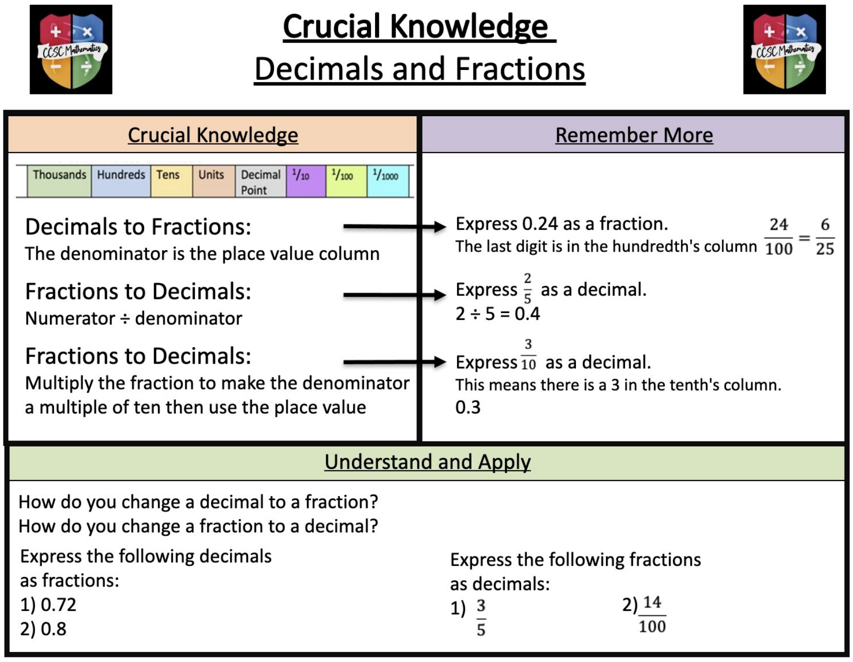
Crucial Knowledge	Remember More
Decimals to Percentages: x100 Move every digit 2 places to the left	Express 0.23 as a percentage. 0.23 x 100 = 23%
Percentages to Decimals: ÷100 Move every digit 2 places to the right	Express 85% as a decimal. 85 ÷ 100 = 0.85

Understand and Apply

How do you change a decimal to a percentage?
How do you change a percentage to a decimal?

Express the following decimals as percentages:	Express the following percentages as decimals:
1) 0.72	1) 64%
2) 0.8	2) 20%
3) 0.04	3) 9%

9 - Converting Decimals and Percentages



Crucial Knowledge
Decimals and Fractions

Crucial Knowledge

Thousands	Hundreds	Tens	Units	Decimal Point	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Decimals to Fractions:
The denominator is the place value column

Fractions to Decimals:
Numerator ÷ denominator

Fractions to Decimals:
Multiply the fraction to make the denominator a multiple of ten then use the place value

Remember More

Express 0.24 as a fraction.
The last digit is in the hundredth's column $\frac{24}{100} = \frac{6}{25}$

Express $\frac{2}{5}$ as a decimal.
2 ÷ 5 = 0.4

Express $\frac{3}{10}$ as a decimal.
This means there is a 3 in the tenth's column.
0.3

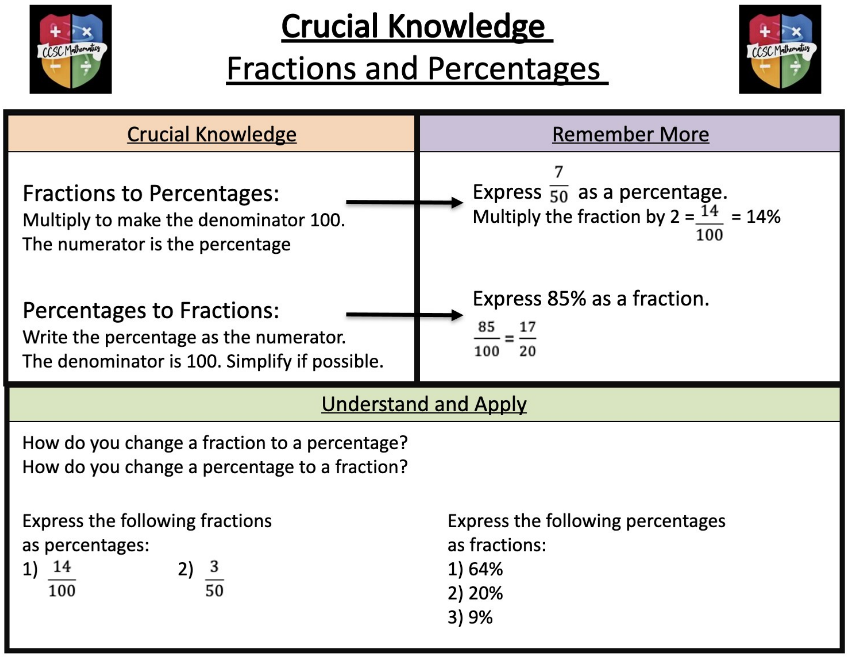
Understand and Apply

How do you change a decimal to a fraction?
How do you change a fraction to a decimal?

Express the following decimals as fractions:
1) 0.72
2) 0.8

Express the following fractions as decimals:
1) $\frac{3}{5}$
2) $\frac{14}{100}$

10 - Converting Decimals and Fractions



Crucial Knowledge
Fractions and Percentages

Crucial Knowledge

Fractions to Percentages:
Multiply to make the denominator 100.
The numerator is the percentage

Percentages to Fractions:
Write the percentage as the numerator.
The denominator is 100. Simplify if possible.

Remember More

Express $\frac{7}{50}$ as a percentage.
Multiply the fraction by 2 = $\frac{14}{100}$ = 14%

Express 85% as a fraction.
$\frac{85}{100} = \frac{17}{20}$

Understand and Apply

How do you change a fraction to a percentage?
How do you change a percentage to a fraction?

Express the following fractions as percentages:
1) $\frac{14}{100}$
2) $\frac{3}{50}$

Express the following percentages as fractions:
1) 64%
2) 20%
3) 9%

11 - Converting Fractions and Percentages

Crucial Knowledge
Fractions of an amount

Crucial Knowledge

Divide the amount by the denominator →

Multiply by the numerator →

Remember More

$\frac{1}{3}$ of 21 = 21 ÷ 3 = 7

$\frac{2}{3}$ of 21 = 21 ÷ 3 = 7, 7 x 2 = 14

Understand and Apply

Which part of the fraction do you divide by?

Which part of the fraction do you multiply by?

Find:

$\frac{1}{4}$ of 24

$\frac{1}{5}$ of 35

$\frac{1}{7}$ of 42

Find:

$\frac{2}{3}$ of 18

$\frac{3}{5}$ of 45

$\frac{4}{7}$ of 49

12 - Fractions of Amounts

Crucial Knowledge
Equivalent Fractions

Crucial Knowledge

Equivalent fractions are two fractions that are equal. You can multiply or divide.

Multiply numerator and denominator by the same number →

Divide numerator and denominator by the same number →

Remember More

$$\frac{1}{3} \xrightarrow[x2]{x2} \frac{2}{6}$$

$$\frac{9}{24} \xrightarrow[÷3]{÷3} \frac{3}{8}$$

Understand and Apply

Fill in the missing numbers:

$\frac{2}{5} = \frac{\quad}{15}$

$\frac{3}{8} = \frac{9}{\quad}$

$\frac{2}{7} = \frac{\quad}{35}$

Fill in the missing numbers:

$\frac{4}{8} = \frac{\quad}{4}$

$\frac{12}{36} = \frac{4}{\quad}$

$\frac{15}{25} = \frac{\quad}{5}$

Give 2 equivalent fractions to the following:

$\frac{1}{4} =$

$\frac{3}{15} =$

$\frac{6}{10} =$

13 - Equivalent Fractions

Crucial Knowledge

Simplifying Fractions

Crucial Knowledge	Remember More
Divide numerator and denominator by the same number. Repeat.	$\frac{10 \xrightarrow{\div 2} 5 \xrightarrow{\div 5} 1}{30 \xrightarrow[\div 2]{} 15 \xrightarrow[\div 5]{} 3}$
Divide numerator and denominator by the highest common factor	$\frac{18 \xrightarrow{\div 6} 3}{30 \xrightarrow[\div 6]{} 5}$

Understand and Apply

Simplify the following fractions

$\frac{3}{15} = \frac{1}{\square}$

$\frac{4}{12} = \frac{\square}{3}$

$\frac{4}{38} = \frac{2}{\square}$

Simplify the following fractions

$\frac{3}{35} =$

$\frac{8}{40} =$

$\frac{4}{64} =$

14 - Simplifying Fractions

Crucial Knowledge

Adding Fractions

Crucial Knowledge	Remember More
Adding Make sure the bottom numbers (the **denominators**) are the same	$\frac{1}{3} + \frac{1}{5}$ $\quad \frac{1}{3} \underset{\times 5}{\overset{\times 5}{=}} \frac{5}{15}$ $\quad \frac{1}{5} \underset{\times 3}{\overset{\times 3}{=}} \frac{3}{15}$
Add the top numbers (the **numerators**), put that answer over the denominator	$\frac{5}{15} + \frac{3}{15} = \frac{8}{15}$

Understand and Apply

Add $\frac{2}{7}$ and $\frac{3}{7}$

Add $\frac{1}{4}$ and $\frac{3}{8}$

Add $\frac{1}{8}$ and $\frac{5}{12}$

Add $\frac{8}{15}$ and $\frac{3}{10}$

15 - Adding Fractions

Crucial Knowledge
Subtracting Fractions

Crucial Knowledge	Remember More
Subtracting Make sure the bottom numbers (the **denominators**) are the same	$\frac{3}{16} - \frac{1}{8} = \frac{3}{16} - \frac{2}{16}$
Subtract the top numbers (the **numerators**), put that answer over the denominator	$= \frac{1}{16}$

Understand and Apply

1) $\frac{3}{6} - \frac{1}{6}$

2) $\frac{3}{4} - \frac{4}{12} =$

3) $\frac{2}{5} - \frac{4}{15}$

4) $\frac{1}{2} - \frac{3}{8} =$

16 - Subtracting Fractions

Crucial Knowledge
Multiplying Fractions

Crucial Knowledge	Remember More
Multiplying Multiply the top numbers (the numerators) Multiply the bottom numbers (the denominators)	$\frac{4}{5} \times \frac{3}{7} = \frac{4 \times 3}{5 \times 7} = \frac{12}{35}$

Understand and Apply

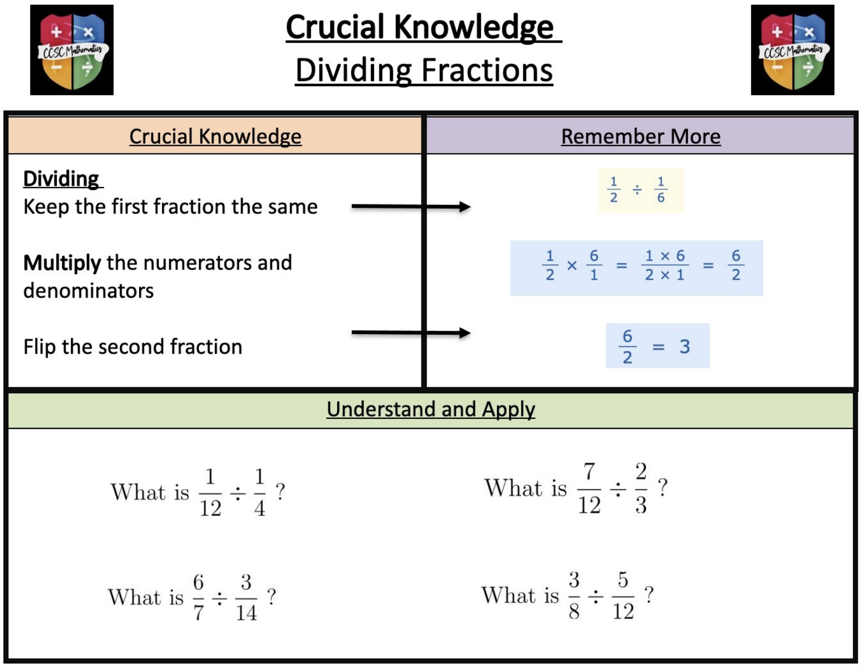
What is $\frac{5}{8} \times \frac{4}{5}$?

What is $\frac{3}{4} \times \frac{5}{6}$?

What is $\frac{3}{7} \times \frac{5}{6}$?

What is $\frac{5}{14} \times \frac{7}{8}$?

17 - Multiplying Fractions



Crucial Knowledge
Dividing Fractions

Crucial Knowledge	Remember More
Dividing Keep the first fraction the same **Multiply** the numerators and denominators Flip the second fraction	$\frac{1}{2} \div \frac{1}{6}$ $\frac{1}{2} \times \frac{6}{1} = \frac{1 \times 6}{2 \times 1} = \frac{6}{2}$ $\frac{6}{2} = 3$

Understand and Apply

What is $\frac{1}{12} \div \frac{1}{4}$?

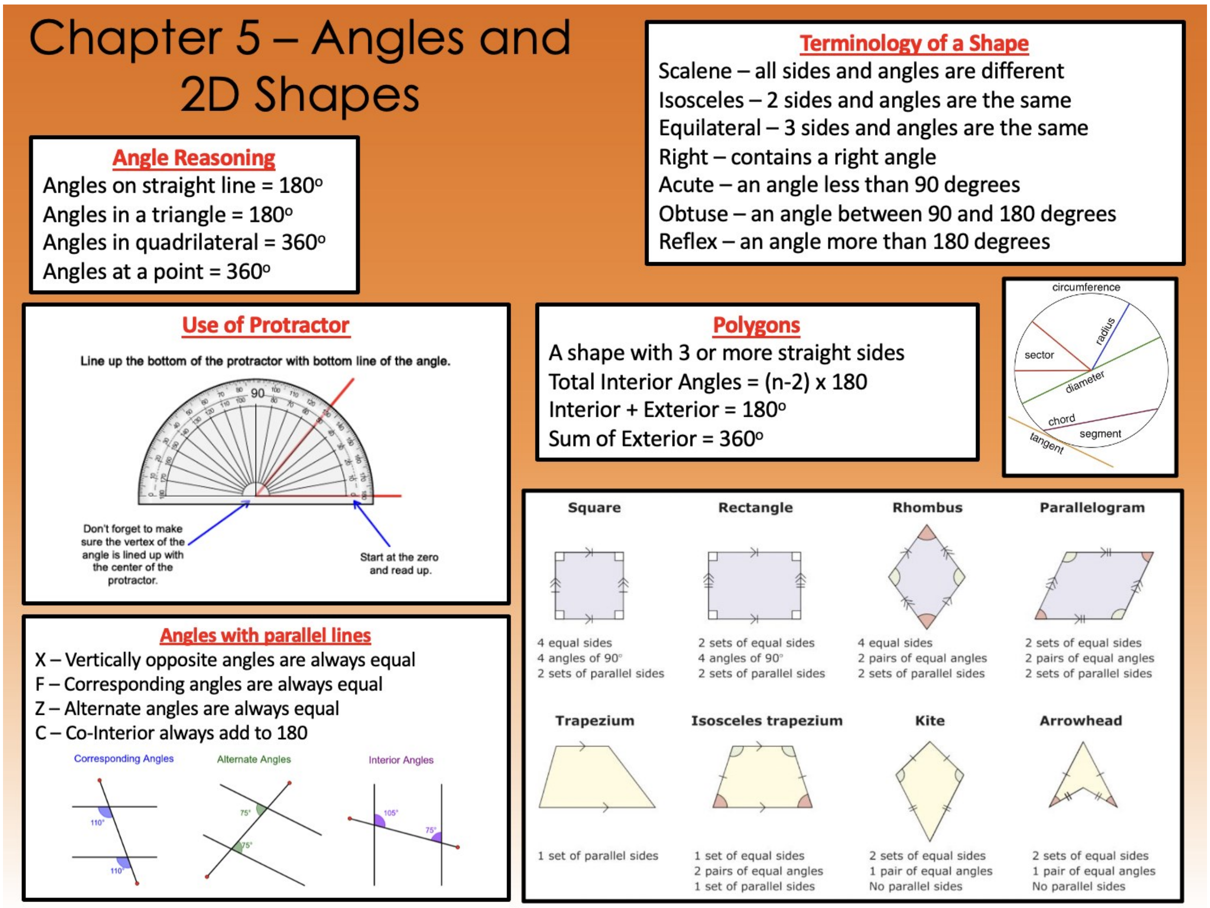
What is $\frac{7}{12} \div \frac{2}{3}$?

What is $\frac{6}{7} \div \frac{3}{14}$?

What is $\frac{3}{8} \div \frac{5}{12}$?

18 - Dividing Fractions

Angles and 2D Shapes



Chapter 5 – Angles and 2D Shapes

Angle Reasoning

Angles on straight line = 180°

Angles in a triangle = 180°

Angles in quadrilateral = 360°

Angles at a point = 360°

Terminology of a Shape

Scalene – all sides and angles are different

Isosceles – 2 sides and angles are the same

Equilateral – 3 sides and angles are the same

Right – contains a right angle

Acute – an angle less than 90 degrees

Obtuse – an angle between 90 and 180 degrees

Reflex – an angle more than 180 degrees

Use of Protractor

Line up the bottom of the protractor with bottom line of the angle.

Don't forget to make sure the vertex of the angle is lined up with the center of the protractor.

Start at the zero and read up.

Polygons

A shape with 3 or more straight sides

Total Interior Angles = (n-2) x 180

Interior + Exterior = 180°

Sum of Exterior = 360°

Angles with parallel lines

X – Vertically opposite angles are always equal

F – Corresponding angles are always equal

Z – Alternate angles are always equal

C – Co-Interior always add to 180

Square
4 equal sides
4 angles of 90°
2 sets of parallel sides

Rectangle
2 sets of equal sides
4 angles of 90°
2 sets of parallel sides

Rhombus
4 equal sides
2 pairs of equal angles
2 sets of parallel sides

Parallelogram
2 sets of equal sides
2 pairs of equal angles
2 sets of parallel sides

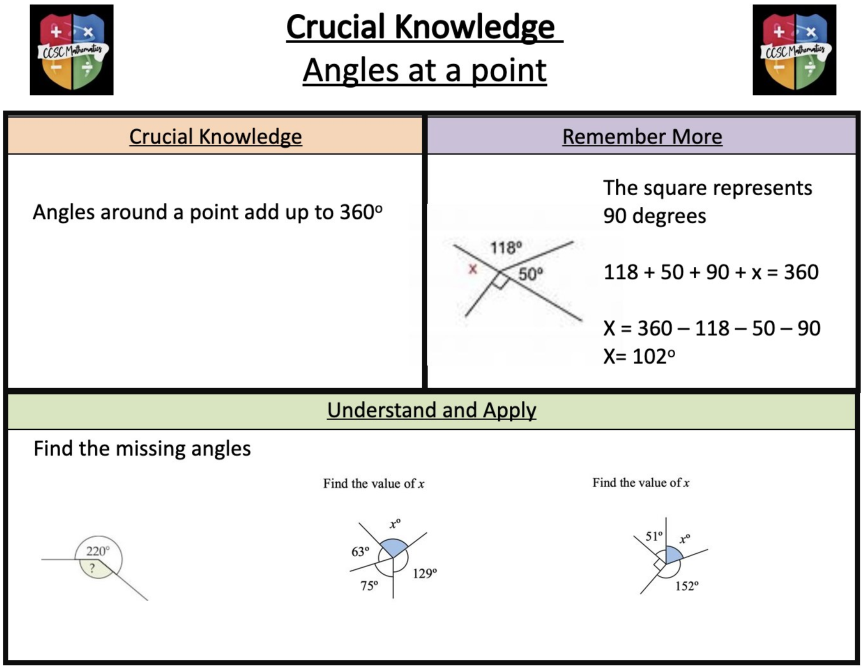
Trapezium
1 set of parallel sides

Isosceles trapezium
1 set of equal sides
2 pairs of equal angles
1 set of parallel sides

Kite
2 sets of equal sides
1 pair of equal angles
No parallel sides

Arrowhead
2 sets of equal sides
1 pair of equal angles
No parallel sides

Know More, Remember More, Understand and Apply



Crucial Knowledge
Angles at a point

Crucial Knowledge

Angles around a point add up to 360°

Remember More

The square represents 90 degrees

118 + 50 + 90 + x = 360

X = 360 – 118 – 50 – 90
X= 102°

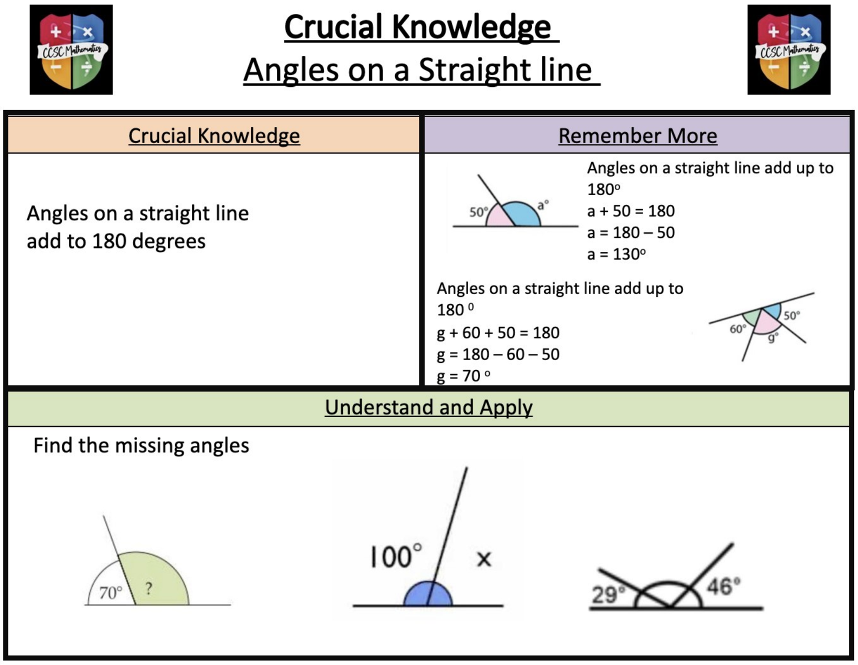
Understand and Apply

Find the missing angles

Find the value of x

Find the value of x

19 - Angles around a point



Crucial Knowledge
Angles on a Straight line

Crucial Knowledge

Angles on a straight line add to 180 degrees

Remember More

Angles on a straight line add up to 180°
a + 50 = 180
a = 180 – 50
a = 130°

Angles on a straight line add up to 180 °
g + 60 + 50 = 180
g = 180 – 60 – 50
g = 70 °

Understand and Apply

Find the missing angles

20 - Angles on a straight line

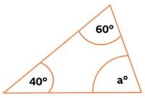
Crucial Knowledge

Angles in triangles

Crucial Knowledge

Angles in a triangle add up to 180°

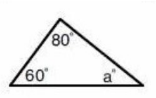
Remember More

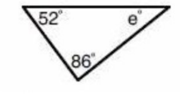


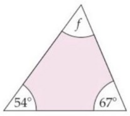
60 + 40 + a = 180
a = 180 – 60 – 40
a = 80°

Understand and Apply

Find the missing angles







21 - Angles in a triangle

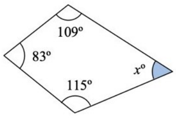
Crucial Knowledge

Angles in a Quadrilateral

Crucial Knowledge

Angles in a quadrilateral add up to 360°

Remember More



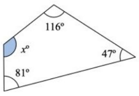
Angles in a Quadrilateral add up to 360°

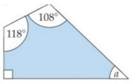
109 + 115 + 83 + x = 360

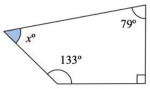
x = 360 – 109 – 115 – 83
x = 53°

Understand and Apply

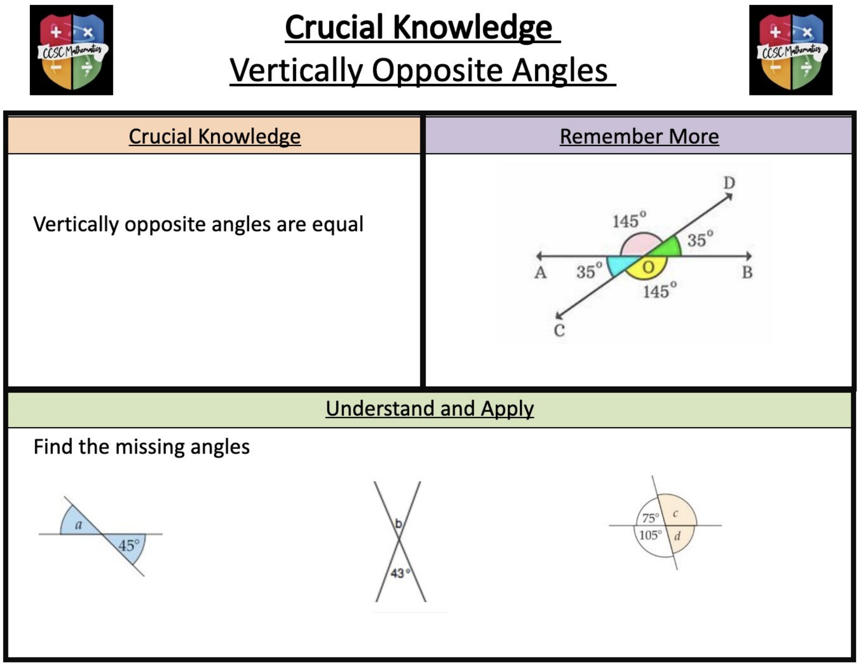
Find the missing angles







22 - Angles in a quadrilateral



Crucial Knowledge
Vertically Opposite Angles

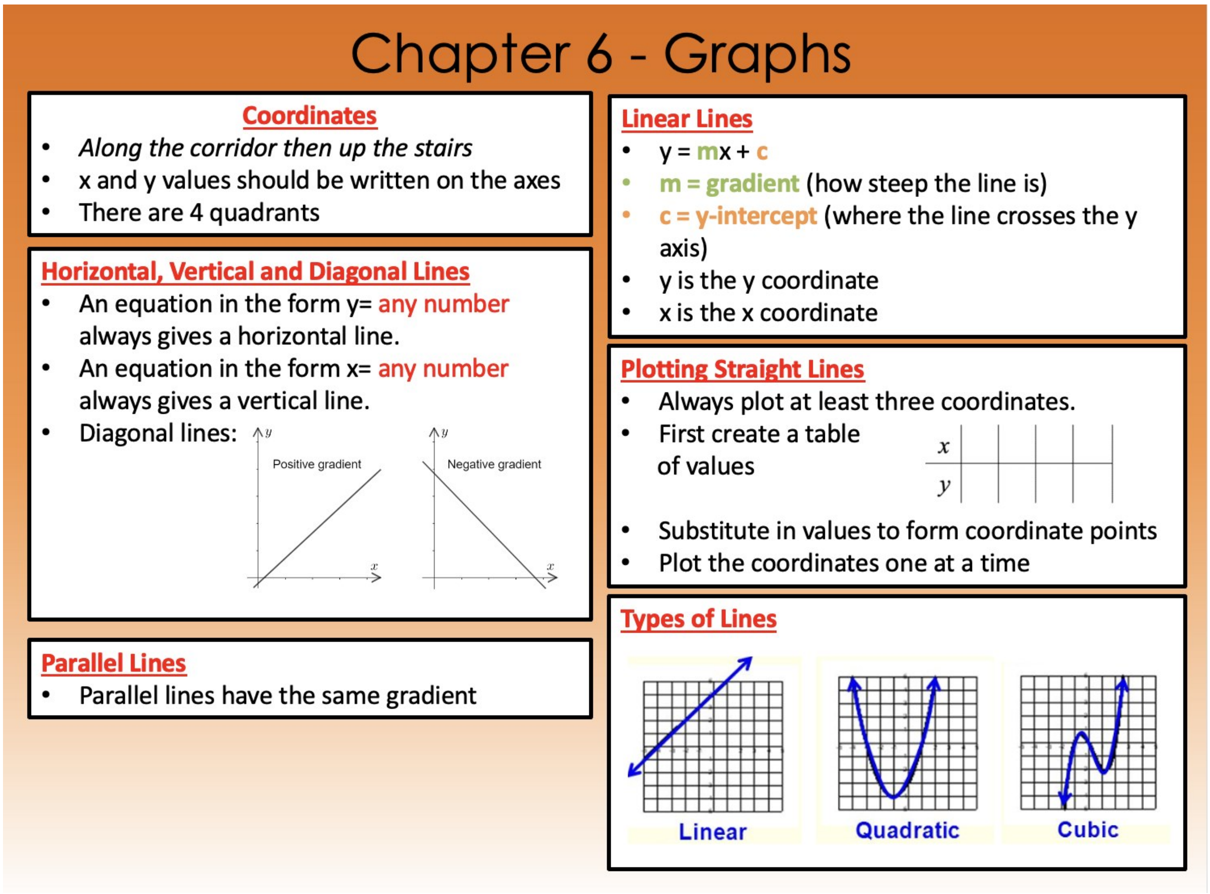
Crucial Knowledge	Remember More
Vertically opposite angles are equal	

Understand and Apply

Find the missing angles

23 - Vertically Opposite

Graphs



Chapter 6 - Graphs

Coordinates

- *Along the corridor then up the stairs*
- x and y values should be written on the axes
- There are 4 quadrants

Horizontal, Vertical and Diagonal Lines

- An equation in the form y= any number always gives a horizontal line.
- An equation in the form x= any number always gives a vertical line.
- Diagonal lines:

Parallel Lines

- Parallel lines have the same gradient

Linear Lines

- $y = mx + c$
- m = gradient (how steep the line is)
- c = y-intercept (where the line crosses the y axis)
- y is the y coordinate
- x is the x coordinate

Plotting Straight Lines

- Always plot at least three coordinates.
- First create a table of values

x				
y				

- Substitute in values to form coordinate points
- Plot the coordinates one at a time

Types of Lines

Chapter 6 – Functions and Graphs

Inverse and Composite Functions

A function is an operation, or set of operations, that is carried out on an input.

A second function could be described as $z=x^2$

A composite function is one where the output of one function forms the input of the second function.

An inverse function is one that returns to the output of the original function to the input.

Perpendicular Lines

Perpendicular means at 90 degrees too.

When finding the gradient of the perpendicular to $y = mx + c$ it will be $-1/m$

Two gradients of two perpendicular lines multiplied together to give -1 or $m \times -1/m = -1$

So if two lines with gradients m and m_2 are perpendicular, then $m_1m_2 = -1$

Exponential Functions

An exponential function has the form:

$Y = ab^x$, $y = ab^{-x}$, or $y = ab^{kx}$

If the power of b is positive the function shows exponential growth

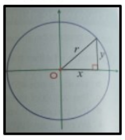
If the power of b is negative the function shows exponential decay

Equation of a Circle

A circle is the locus of points that are a set distance from a fixed point.

In this diagram the fixed point is O.

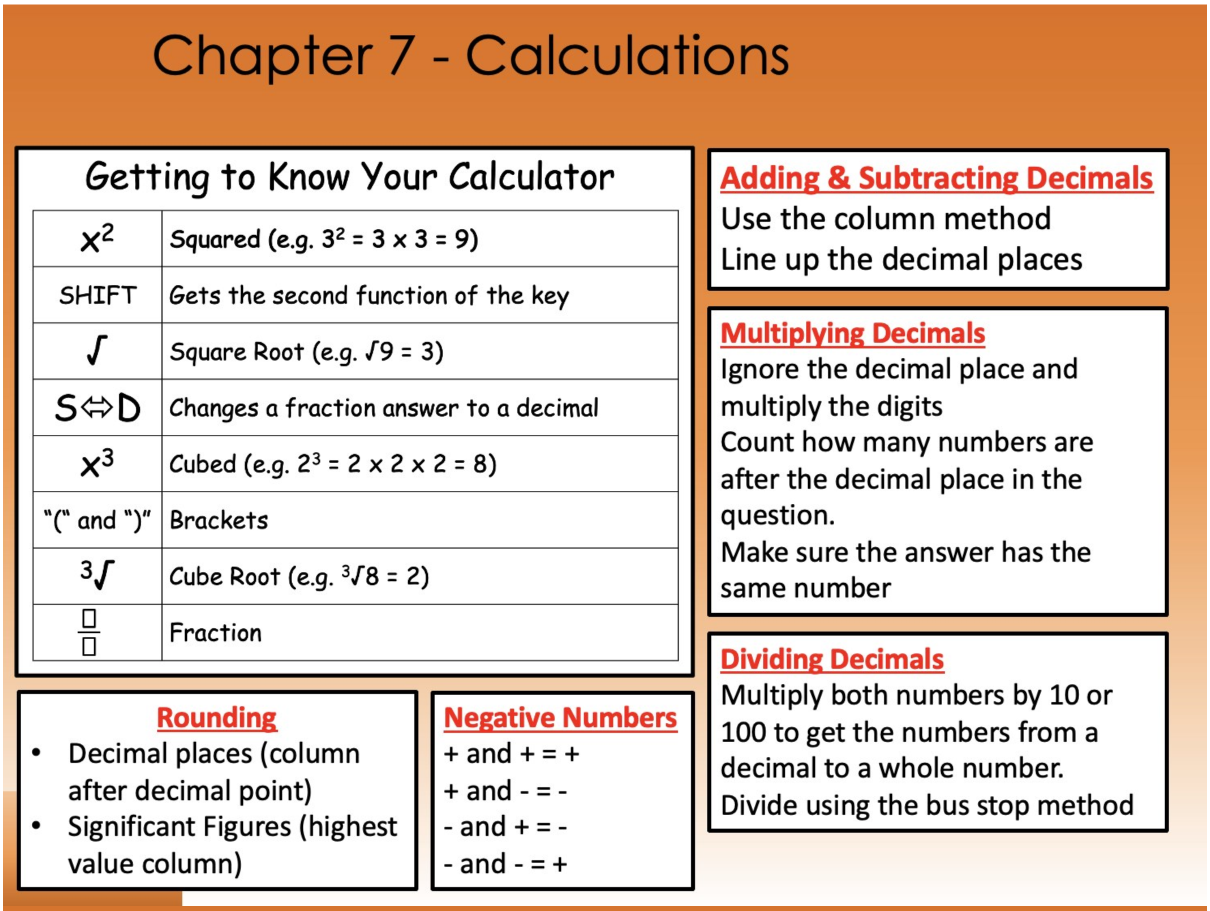
It is the centre of the circle. The radius of the circle is r.



Pythagoras Theorem tells us that $x^2 + y^2 = r^2$.

This is the equation of a circle centre origin and radius r.

Calculations



Chapter 7 - Calculations

Getting to Know Your Calculator

x^2	Squared (e.g. $3^2 = 3 \times 3 = 9$)
SHIFT	Gets the second function of the key
√	Square Root (e.g. √9 = 3)
S⇔D	Changes a fraction answer to a decimal
x^3	Cubed (e.g. $2^3 = 2 \times 2 \times 2 = 8$)
"(" and ")"	Brackets
³√	Cube Root (e.g. ³√8 = 2)
$\frac{\square}{\square}$	Fraction

Adding & Subtracting Decimals

Use the column method

Line up the decimal places

Multiplying Decimals

Ignore the decimal place and multiply the digits

Count how many numbers are after the decimal place in the question.

Make sure the answer has the same number

Dividing Decimals

Multiply both numbers by 10 or 100 to get the numbers from a decimal to a whole number.

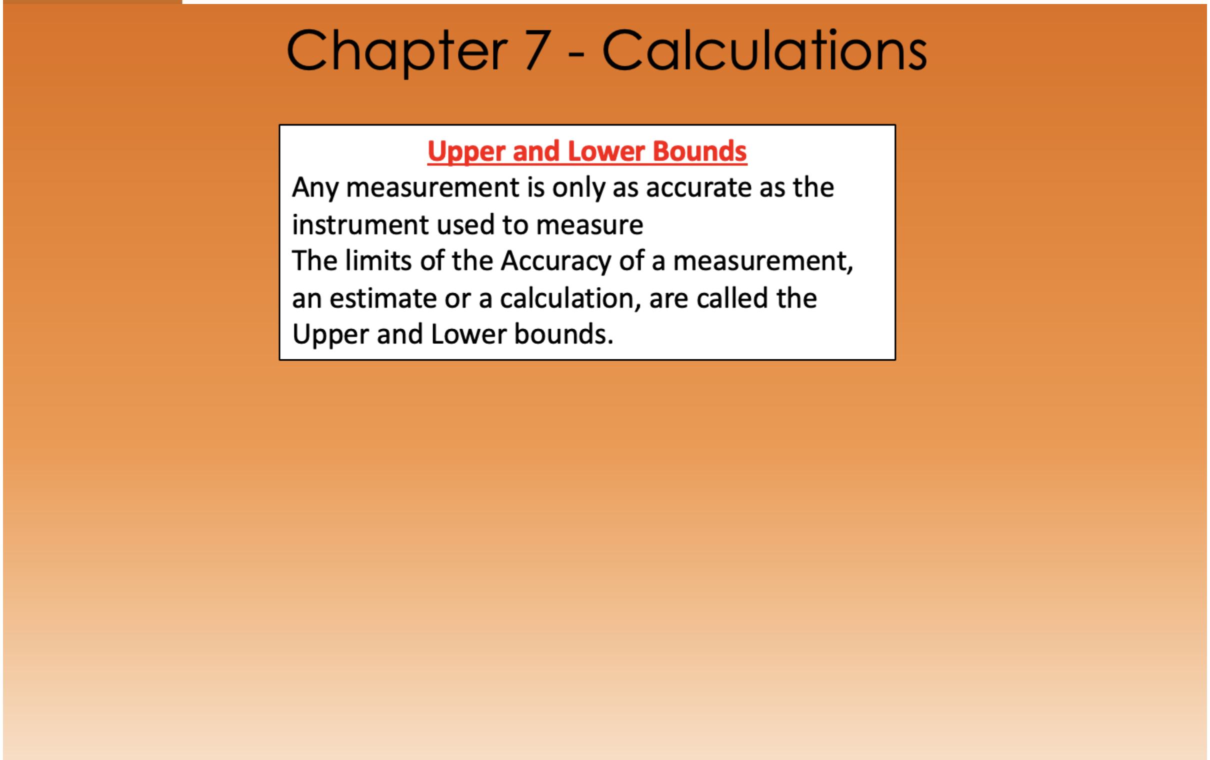
Divide using the bus stop method

Rounding

- Decimal places (column after decimal point)
- Significant Figures (highest value column)

Negative Numbers

+ and + = +
+ and - = -
- and + = -
- and - = +



Chapter 7 - Calculations

Upper and Lower Bounds

Any measurement is only as accurate as the instrument used to measure

The limits of the Accuracy of a measurement, an estimate or a calculation, are called the Upper and Lower bounds.

Know More, Remember More, Understand and Apply

Crucial Knowledge
Multiplying Decimals

Crucial Knowledge	Remember More
Multiply each decimal number by 10, 100 or 1000 to make it a whole number.	0.12 x 0.5 x100 x10
Multiply the whole numbers.	12 x 5 = 60
Divide your answer by the same number that you multiplied the question by.	60 ÷ 100 ÷ 10 = **0.06**

Understand and Apply

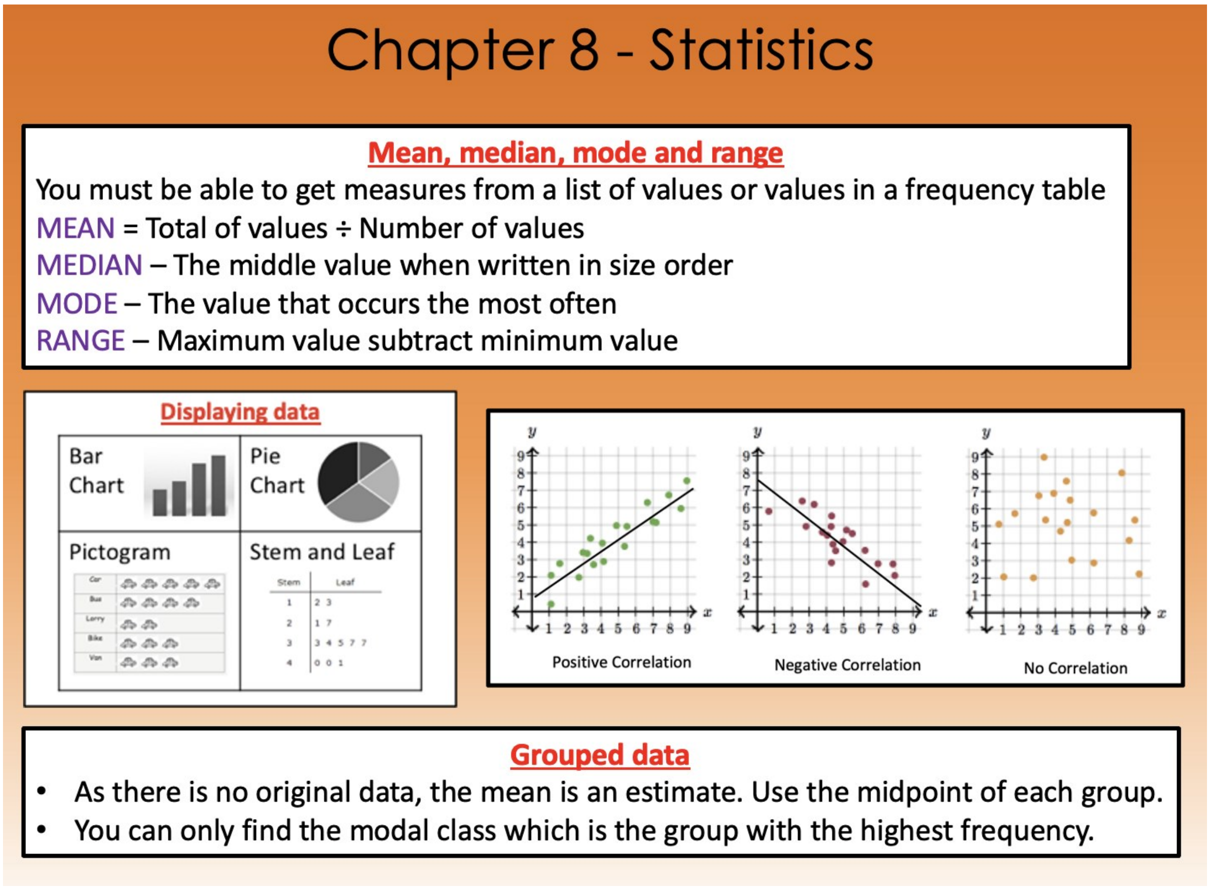
Showing each step of the method, calculate...

a) 0.6 x 4

b) 0.6 x 0.4

c) 0.04 x 0.6

Statistics



Chapter 8 - Statistics

Mean, median, mode and range

You must be able to get measures from a list of values or values in a frequency table

MEAN = Total of values ÷ Number of values

MEDIAN – The middle value when written in size order

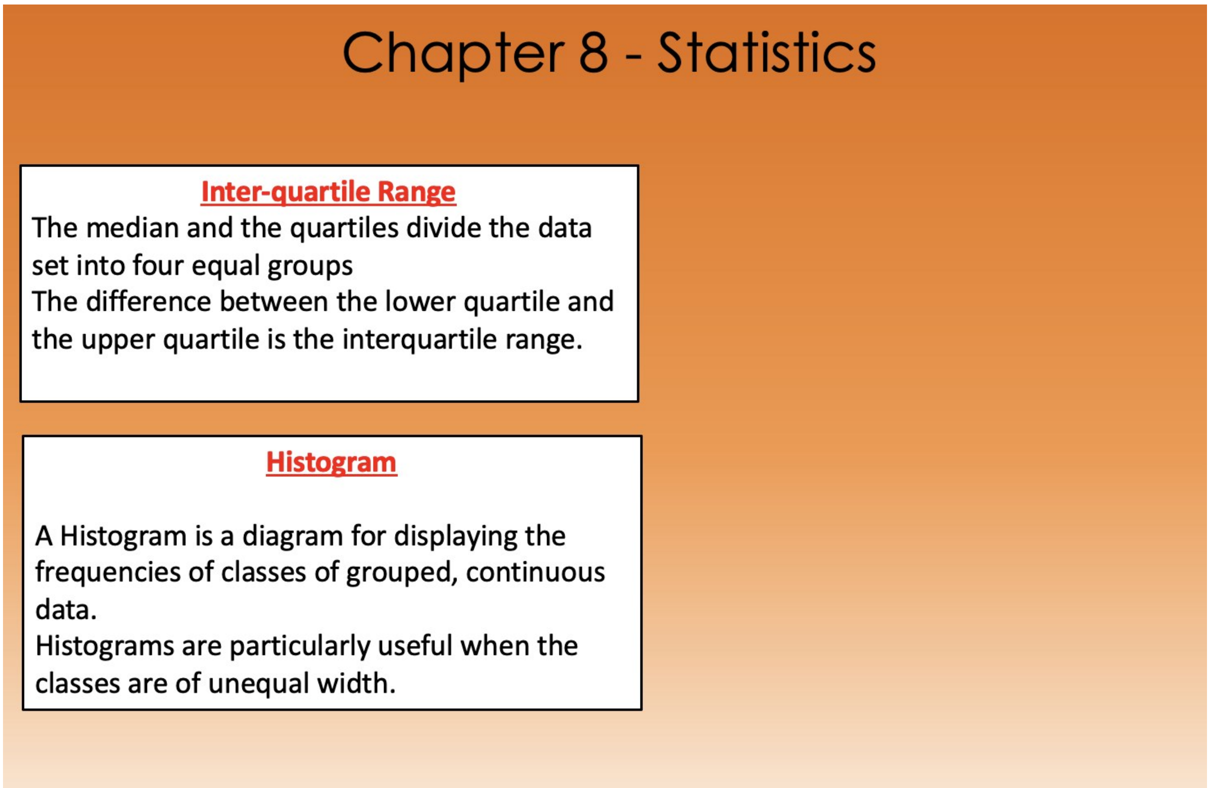
MODE – The value that occurs the most often

RANGE – Maximum value subtract minimum value

Displaying data

Grouped data

- As there is no original data, the mean is an estimate. Use the midpoint of each group.
- You can only find the modal class which is the group with the highest frequency.



Chapter 8 - Statistics

Inter-quartile Range

The median and the quartiles divide the data set into four equal groups

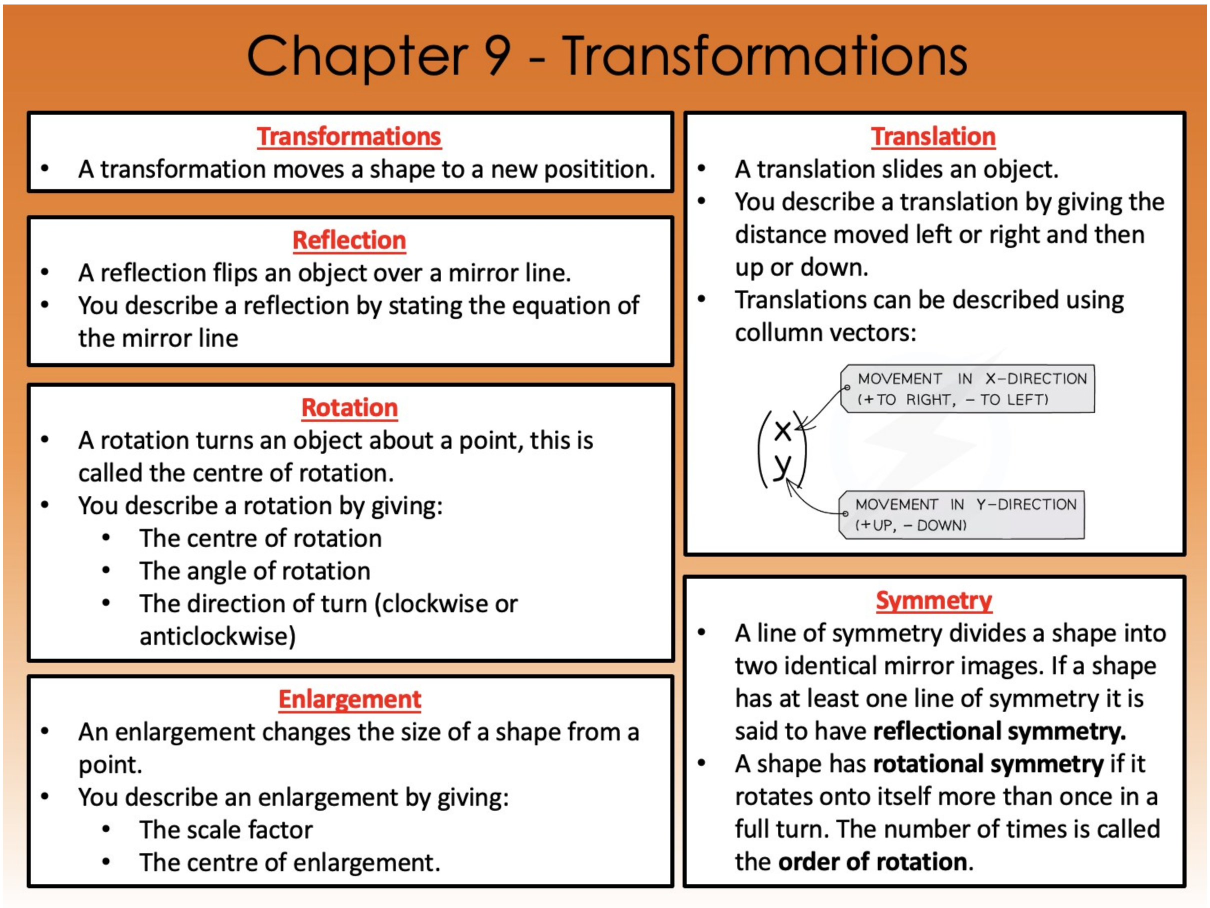
The difference between the lower quartile and the upper quartile is the interquartile range.

Histogram

A Histogram is a diagram for displaying the frequencies of classes of grouped, continuous data.

Histograms are particularly useful when the classes are of unequal width.

Transformations



Chapter 9 - Transformations

Transformations

- A transformation moves a shape to a new positition.

Reflection

- A reflection flips an object over a mirror line.
- You describe a reflection by stating the equation of the mirror line

Rotation

- A rotation turns an object about a point, this is called the centre of rotation.
- You describe a rotation by giving:
  - The centre of rotation
  - The angle of rotation
  - The direction of turn (clockwise or anticlockwise)

Enlargement

- An enlargement changes the size of a shape from a point.
- You describe an enlargement by giving:
  - The scale factor
  - The centre of enlargement.

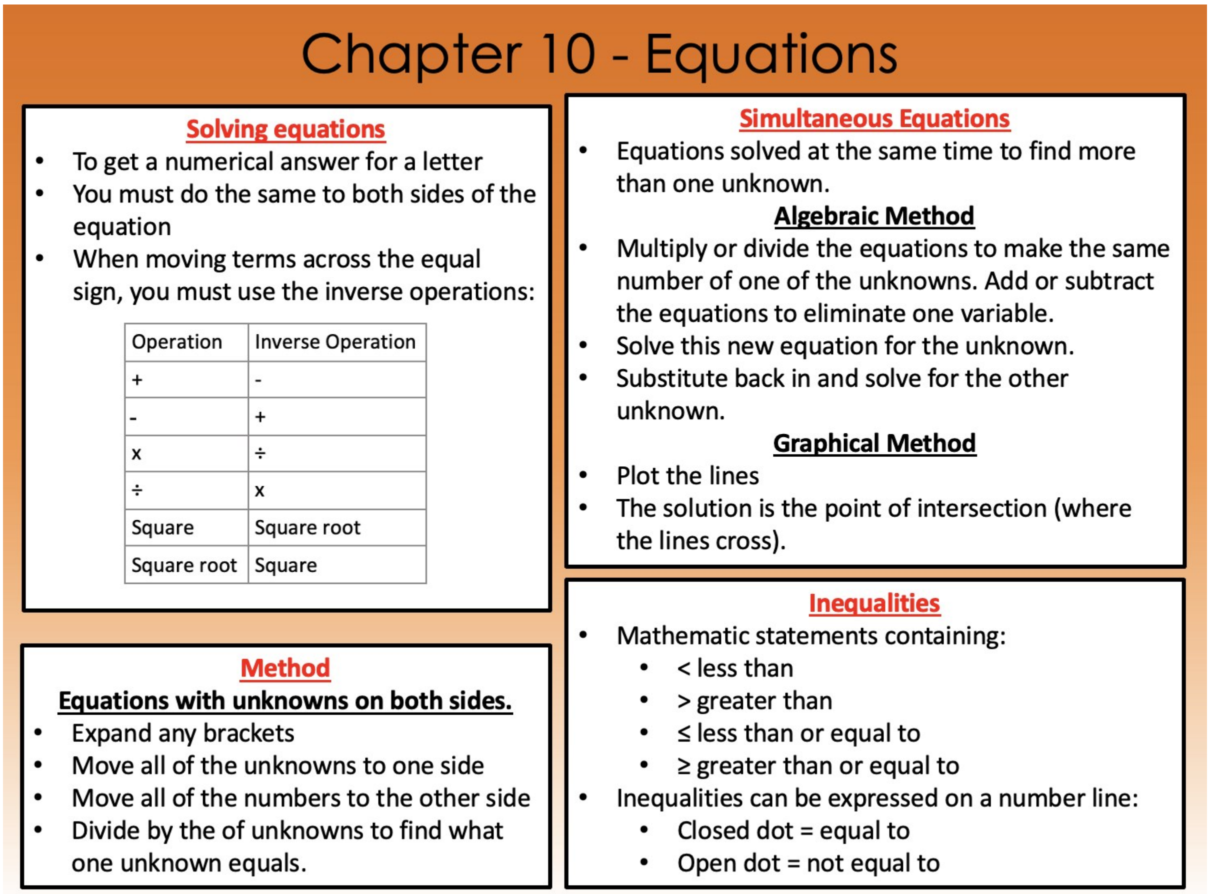
Translation

- A translation slides an object.
- You describe a translation by giving the distance moved left or right and then up or down.
- Translations can be described using collumn vectors:

Symmetry

- A line of symmetry divides a shape into two identical mirror images. If a shape has at least one line of symmetry it is said to have **reflectional symmetry.**
- A shape has **rotational symmetry** if it rotates onto itself more than once in a full turn. The number of times is called the **order of rotation**.

Equations



Chapter 10 - Equations

Solving equations

- To get a numerical answer for a letter
- You must do the same to both sides of the equation
- When moving terms across the equal sign, you must use the inverse operations:

Operation	Inverse Operation
+	-
-	+
x	÷
÷	x
Square	Square root
Square root	Square

Method

Equations with unknowns on both sides.

- Expand any brackets
- Move all of the unknowns to one side
- Move all of the numbers to the other side
- Divide by the of unknowns to find what one unknown equals.

Simultaneous Equations

- Equations solved at the same time to find more than one unknown.

Algebraic Method

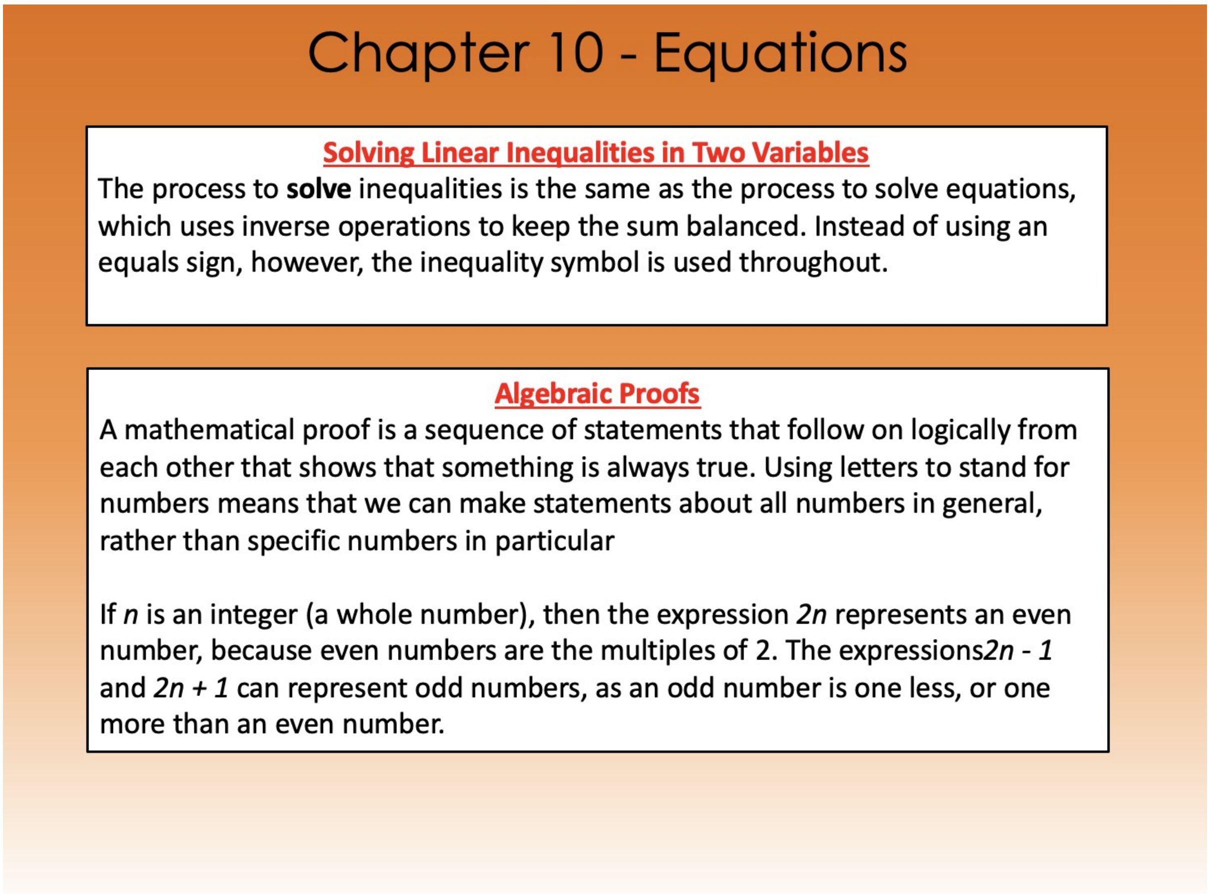
- Multiply or divide the equations to make the same number of one of the unknowns. Add or subtract the equations to eliminate one variable.
- Solve this new equation for the unknown.
- Substitute back in and solve for the other unknown.

Graphical Method

- Plot the lines
- The solution is the point of intersection (where the lines cross).

Inequalities

- Mathematic statements containing:
  - $<$ less than
  - $>$ greater than
  - $\leq$ less than or equal to
  - $\geq$ greater than or equal to
- Inequalities can be expressed on a number line:
  - Closed dot = equal to
  - Open dot = not equal to



Chapter 10 - Equations

Solving Linear Inequalities in Two Variables

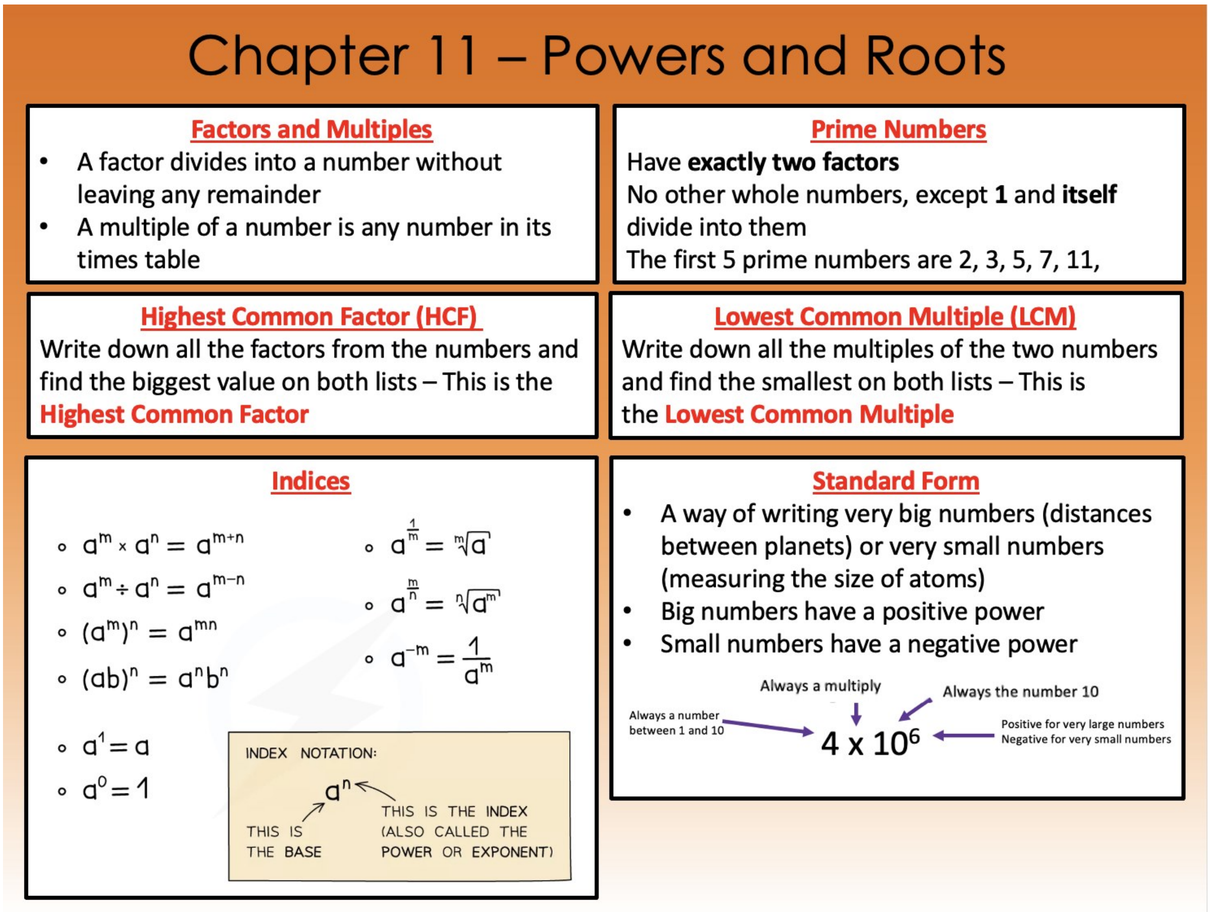
The process to **solve** inequalities is the same as the process to solve equations, which uses inverse operations to keep the sum balanced. Instead of using an equals sign, however, the inequality symbol is used throughout.

Algebraic Proofs

A mathematical proof is a sequence of statements that follow on logically from each other that shows that something is always true. Using letters to stand for numbers means that we can make statements about all numbers in general, rather than specific numbers in particular

If n is an integer (a whole number), then the expression $2n$ represents an even number, because even numbers are the multiples of 2. The expressions$2n - 1$ and $2n + 1$ can represent odd numbers, as an odd number is one less, or one more than an even number.

Powers and Roots



Chapter 11 – Powers and Roots

Factors and Multiples

- A factor divides into a number without leaving any remainder
- A multiple of a number is any number in its times table

Prime Numbers

Have **exactly two factors**

No other whole numbers, except **1** and **itself** divide into them

The first 5 prime numbers are 2, 3, 5, 7, 11,

Highest Common Factor (HCF)

Write down all the factors from the numbers and find the biggest value on both lists – This is the **Highest Common Factor**

Lowest Common Multiple (LCM)

Write down all the multiples of the two numbers and find the smallest on both lists – This is the **Lowest Common Multiple**

Indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $a^1 = a$
- $a^0 = 1$
- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $a^{-m} = \frac{1}{a^m}$

INDEX NOTATION:

a^n

THIS IS THE BASE

THIS IS THE INDEX (ALSO CALLED THE POWER OR EXPONENT)

Standard Form

- A way of writing very big numbers (distances between planets) or very small numbers (measuring the size of atoms)
- Big numbers have a positive power
- Small numbers have a negative power

Always a multiply

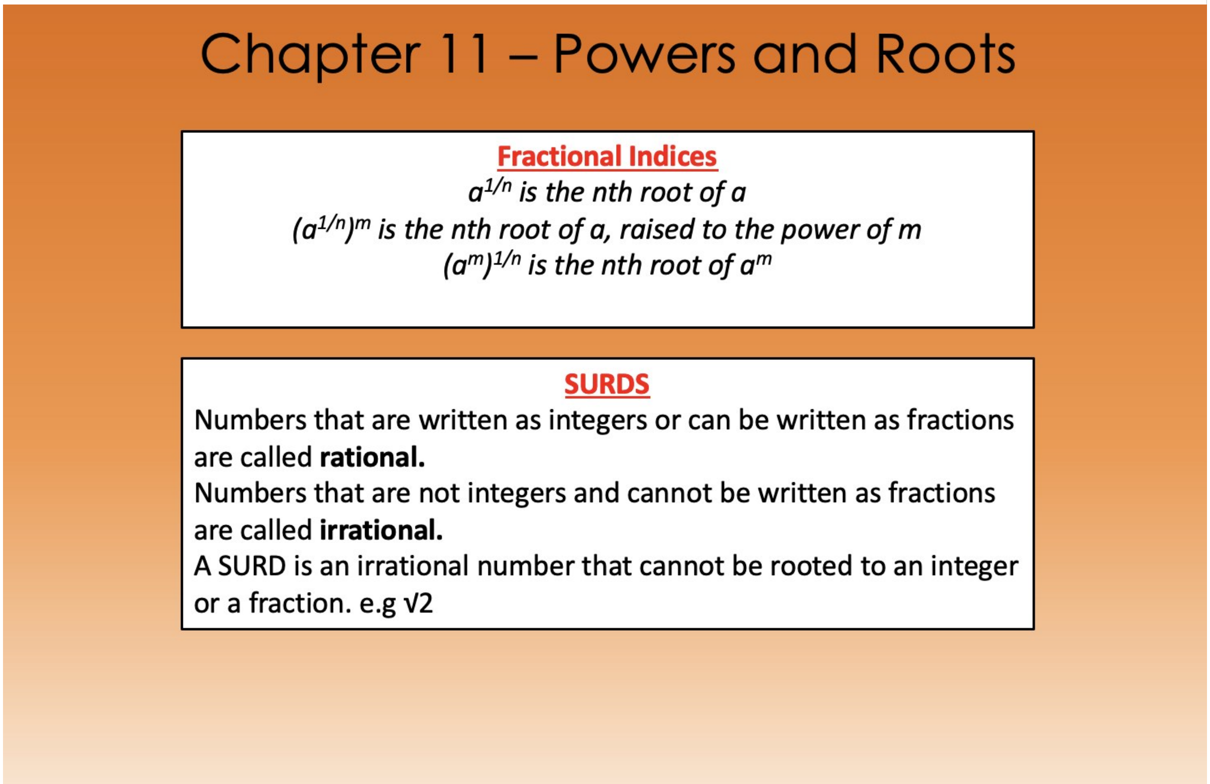
Always the number 10

Always a number between 1 and 10

4×10^6

Positive for very large numbers

Negative for very small numbers



Chapter 11 – Powers and Roots

Fractional Indices

$a^{1/n}$ *is the nth root of a*

$(a^{1/n})^m$ *is the nth root of a, raised to the power of m*

$(a^m)^{1/n}$ *is the nth root of* a^m

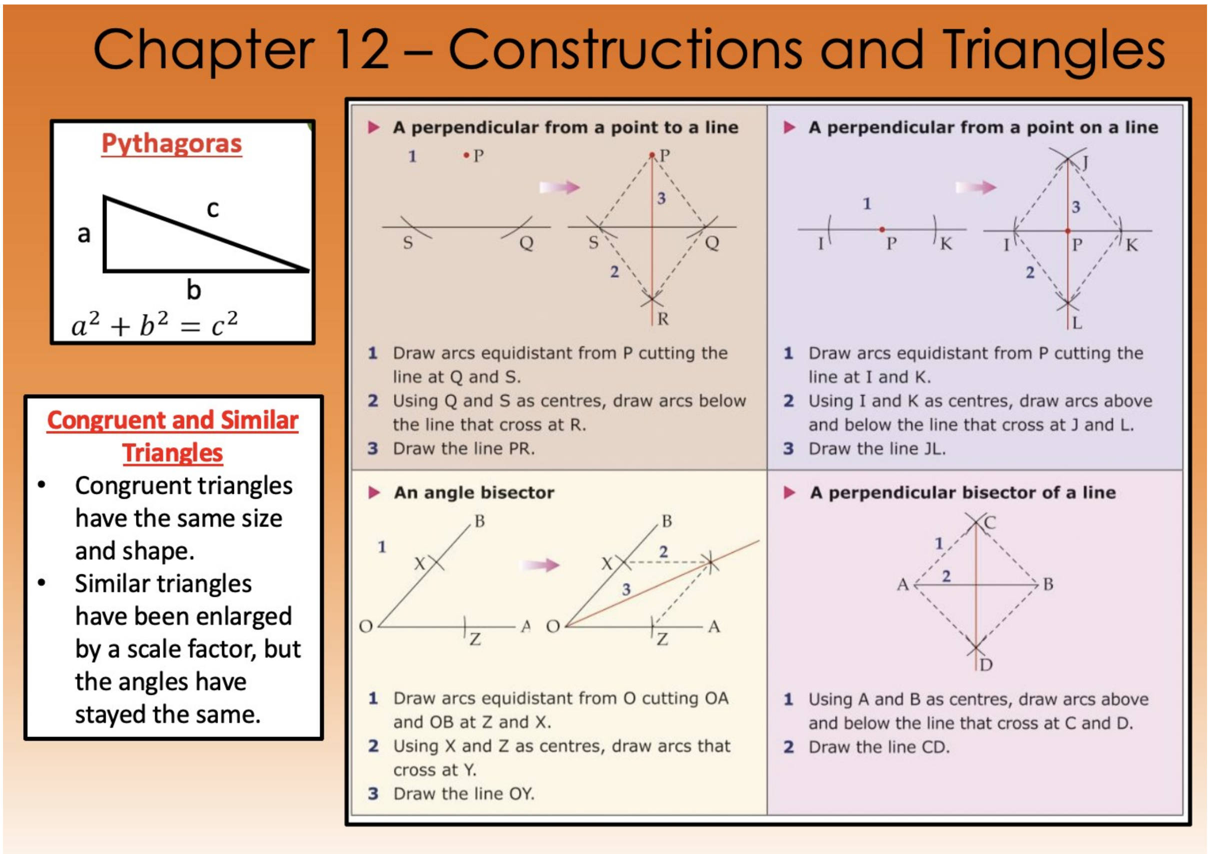
SURDS

Numbers that are written as integers or can be written as fractions are called **rational.**

Numbers that are not integers and cannot be written as fractions are called **irrational.**

A SURD is an irrational number that cannot be rooted to an integer or a fraction. e.g √2

Constructions and Triangles



Chapter 12 – Constructions and Triangles

Pythagoras

$a^2 + b^2 = c^2$

Congruent and Similar Triangles

- Congruent triangles have the same size and shape.
- Similar triangles have been enlarged by a scale factor, but the angles have stayed the same.

A perpendicular from a point to a line

1 Draw arcs equidistant from P cutting the line at Q and S.
2 Using Q and S as centres, draw arcs below the line that cross at R.
3 Draw the line PR.

A perpendicular from a point on a line

1 Draw arcs equidistant from P cutting the line at I and K.
2 Using I and K as centres, draw arcs above and below the line that cross at J and L.
3 Draw the line JL.

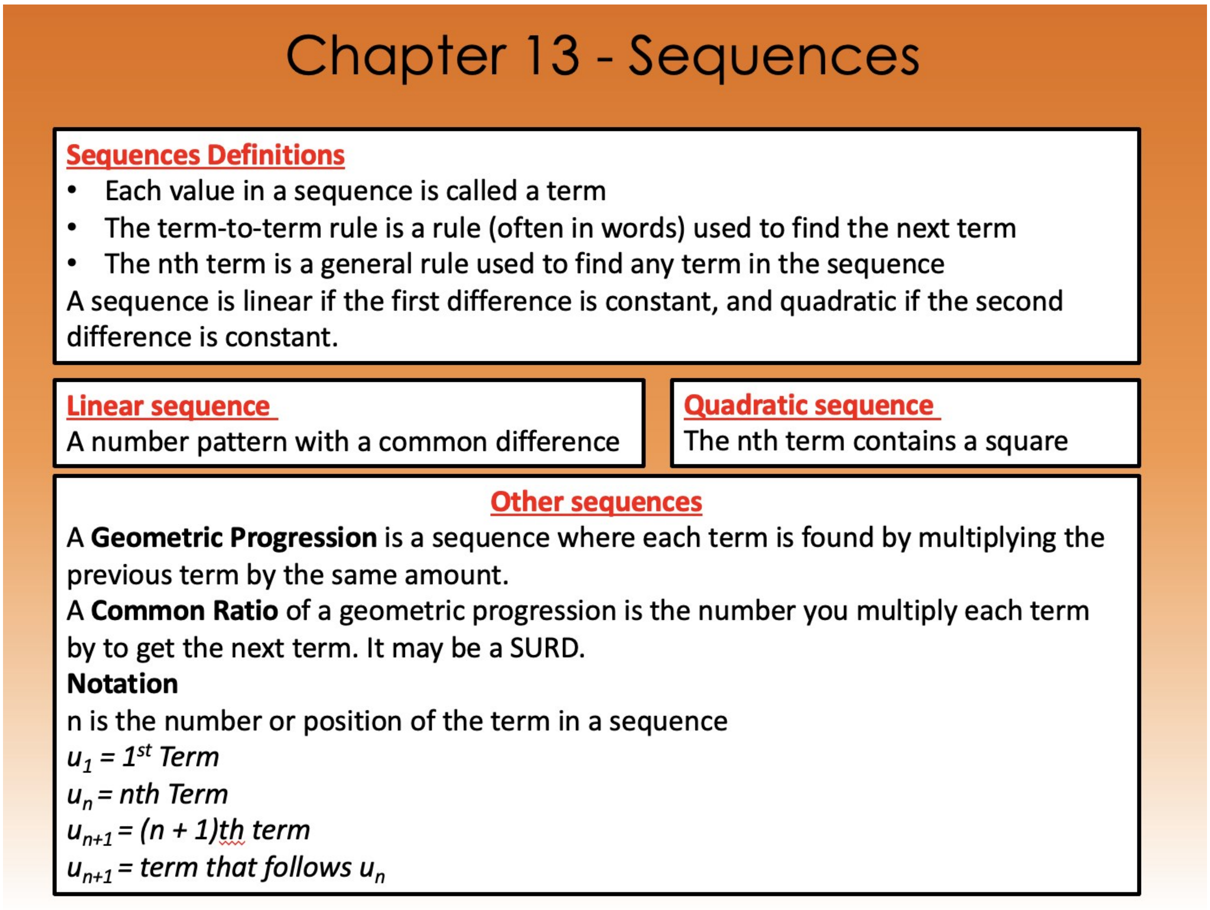
An angle bisector

1 Draw arcs equidistant from O cutting OA and OB at Z and X.
2 Using X and Z as centres, draw arcs that cross at Y.
3 Draw the line OY.

A perpendicular bisector of a line

1 Using A and B as centres, draw arcs above and below the line that cross at C and D.
2 Draw the line CD.

Sequences



Chapter 13 - Sequences

Sequences Definitions

- Each value in a sequence is called a term
- The term-to-term rule is a rule (often in words) used to find the next term
- The nth term is a general rule used to find any term in the sequence

A sequence is linear if the first difference is constant, and quadratic if the second difference is constant.

Linear sequence

A number pattern with a common difference

Quadratic sequence

The nth term contains a square

Other sequences

A **Geometric Progression** is a sequence where each term is found by multiplying the previous term by the same amount.

A **Common Ratio** of a geometric progression is the number you multiply each term by to get the next term. It may be a SURD.

Notation

n is the number or position of the term in a sequence

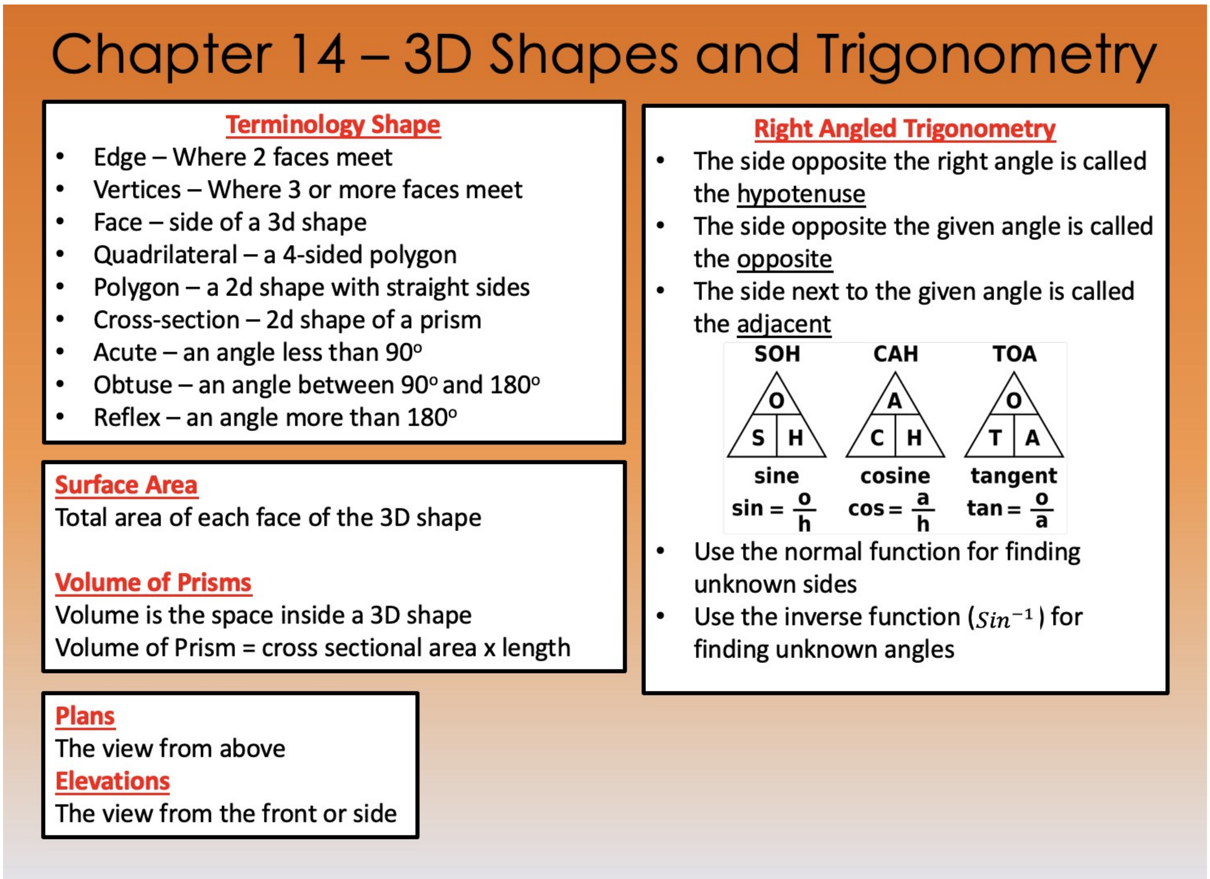
$u_1 = 1^{st}$ *Term*

$u_n =$ *nth Term*

$u_{n+1} = (n + 1)$*th term*

$u_{n+1} =$ *term that follows* u_n

3D shapes and Trigonometry



Chapter 14 – 3D Shapes and Trigonometry

Terminology Shape

- Edge – Where 2 faces meet
- Vertices – Where 3 or more faces meet
- Face – side of a 3d shape
- Quadrilateral – a 4-sided polygon
- Polygon – a 2d shape with straight sides
- Cross-section – 2d shape of a prism
- Acute – an angle less than 90°
- Obtuse – an angle between 90° and 180°
- Reflex – an angle more than 180°

Surface Area

Total area of each face of the 3D shape

Volume of Prisms

Volume is the space inside a 3D shape

Volume of Prism = cross sectional area x length

Plans

The view from above

Elevations

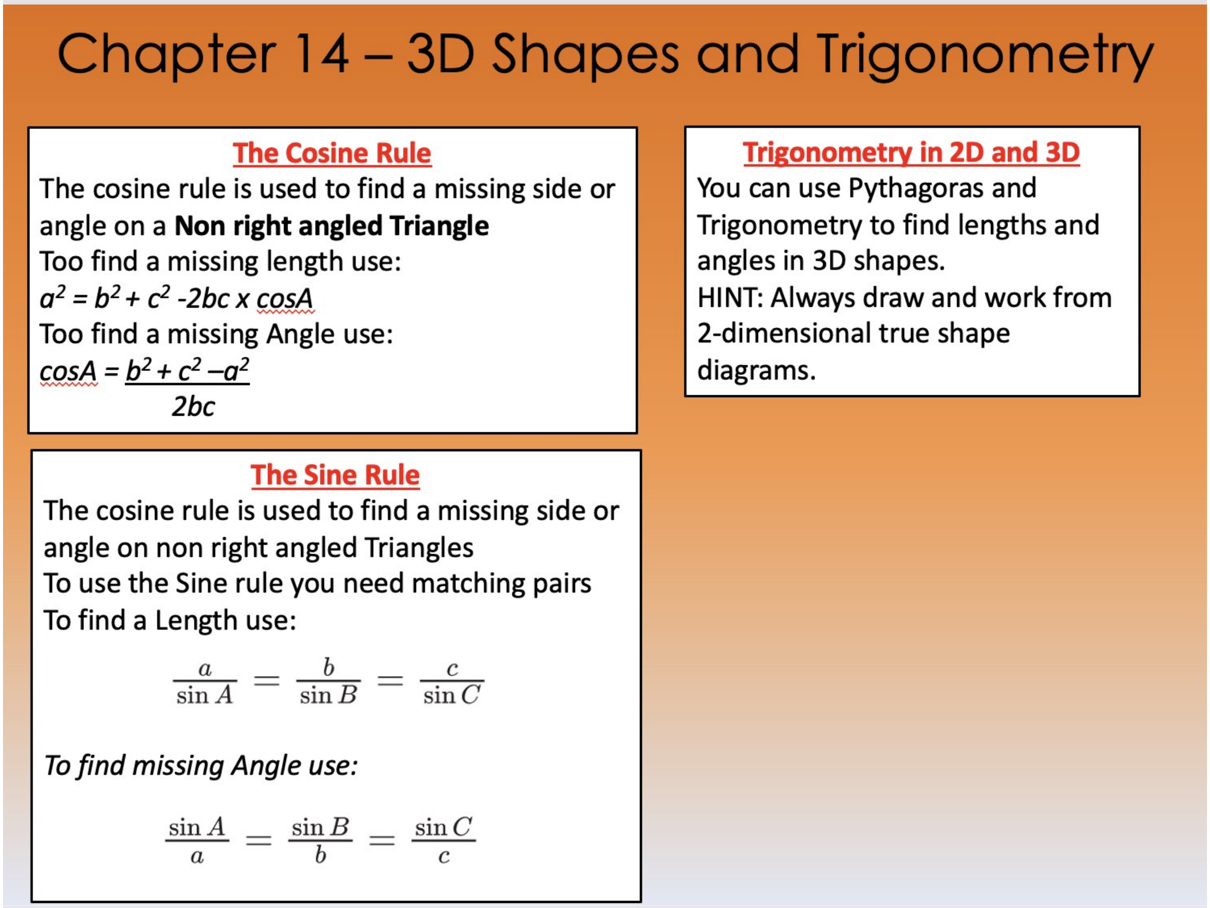
The view from the front or side

Right Angled Trigonometry

- The side opposite the right angle is called the hypotenuse
- The side opposite the given angle is called the opposite
- The side next to the given angle is called the adjacent

$\sin = \frac{o}{h}$ $\cos = \frac{a}{h}$ $\tan = \frac{o}{a}$

- Use the normal function for finding unknown sides
- Use the inverse function (Sin^{-1}) for finding unknown angles



Chapter 14 – 3D Shapes and Trigonometry

The Cosine Rule

The cosine rule is used to find a missing side or angle on a **Non right angled Triangle**

Too find a missing length use:

$a^2 = b^2 + c^2 -2bc \text{ x } cosA$

Too find a missing Angle use:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Trigonometry in 2D and 3D

You can use Pythagoras and Trigonometry to find lengths and angles in 3D shapes.

HINT: Always draw and work from 2-dimensional true shape diagrams.

The Sine Rule

The cosine rule is used to find a missing side or angle on non right angled Triangles

To use the Sine rule you need matching pairs

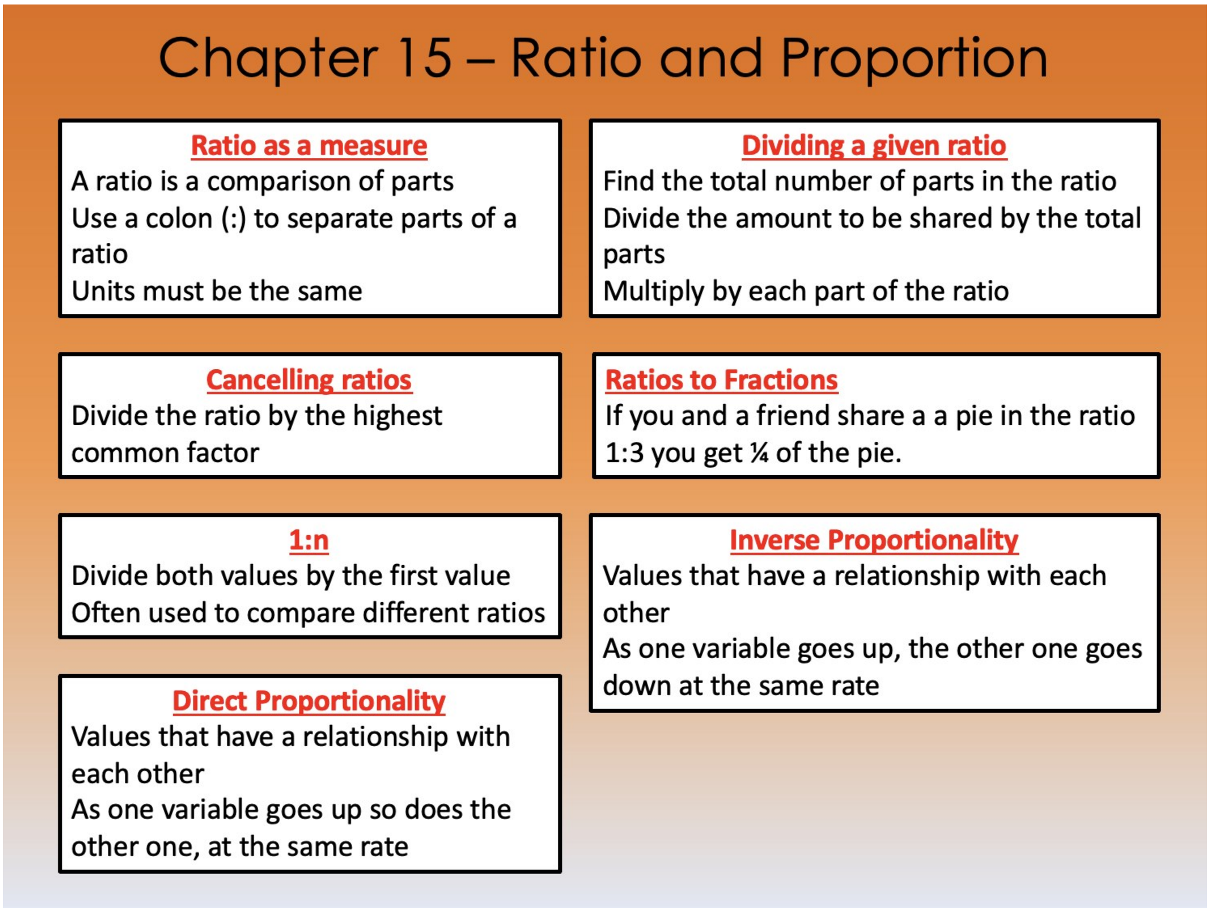
To find a Length use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To find missing Angle use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ratio and Proportion



Chapter 15 – Ratio and Proportion

Ratio as a measure

A ratio is a comparison of parts
Use a colon (:) to separate parts of a ratio
Units must be the same

Dividing a given ratio

Find the total number of parts in the ratio
Divide the amount to be shared by the total parts
Multiply by each part of the ratio

Cancelling ratios

Divide the ratio by the highest common factor

Ratios to Fractions

If you and a friend share a a pie in the ratio 1:3 you get ¼ of the pie.

1:n

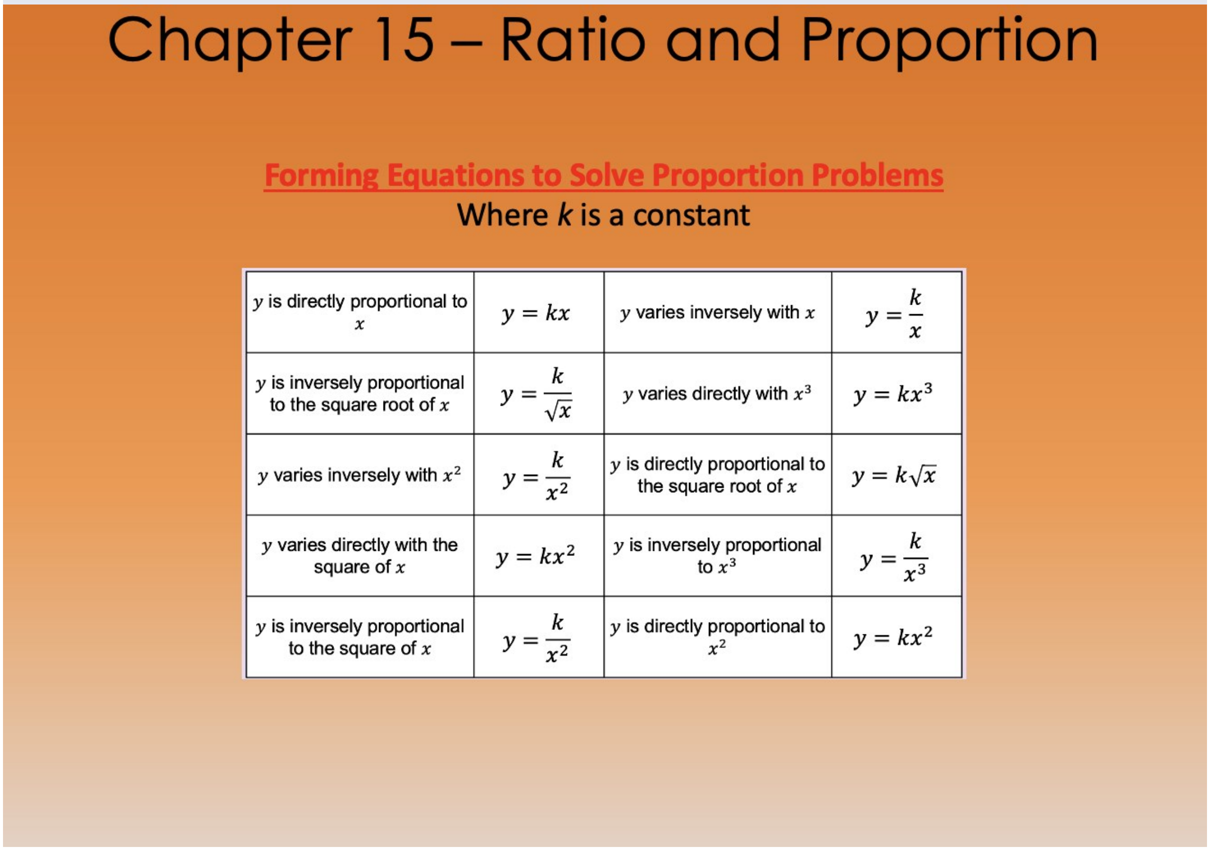
Divide both values by the first value
Often used to compare different ratios

Inverse Proportionality

Values that have a relationship with each other
As one variable goes up, the other one goes down at the same rate

Direct Proportionality

Values that have a relationship with each other
As one variable goes up so does the other one, at the same rate



Chapter 15 – Ratio and Proportion

Forming Equations to Solve Proportion Problems

Where k is a constant

y is directly proportional to x	$y = kx$	y varies inversely with x	$y = \frac{k}{x}$
y is inversely proportional to the square root of x	$y = \frac{k}{\sqrt{x}}$	y varies directly with x^3	$y = kx^3$
y varies inversely with x^2	$y = \frac{k}{x^2}$	y is directly proportional to the square root of x	$y = k\sqrt{x}$
y varies directly with the square of x	$y = kx^2$	y is inversely proportional to x^3	$y = \frac{k}{x^3}$
y is inversely proportional to the square of x	$y = \frac{k}{x^2}$	y is directly proportional to x^2	$y = kx^2$

Know More, Remember More, Understand and Apply

Crucial Knowledge
Simplifying a Ratio

Crucial Knowledge	Remember More
Many ratios can be simplified by finding the **largest** number that divides **into** the numbers in the ratio	The ratio of red to blue dinosaurs is **12:18**. This ratio can be simplified. Both 12 and 18 can be divided by 6. **12 ÷ 6 = 2** **18 ÷ 6 = 3** So a simpler way to write 12:18 is **2:3**

Understand and Apply

Simplify the following ratios:

a) 12:15 b) 25:20 c) 35:21 d) 18:30 e) 18:26

Crucial Knowledge
Sharing in a Ratio

Crucial Knowledge	Remember More
Amounts can be shared in a ratio. £250 is shared in the ratio 3:2 This means that the money is split into **5 parts** (3 parts for one person and 2 parts for the other)	Each part will be worth £50 (£250 ÷ **5 parts**) The person that receives 3 parts will get 3 x £50 = **£150** The person that receives 2 parts will get 2 x £50 = **£100**

Understand and Apply

1) Share £350 in the ratio 2:5

2) Share £450 in the ratio 5:4

Crucial Knowledge
Writing a Ratio

Crucial Knowledge	Remember More
A ratio shows how much of one thing there is compared to another.	If you are making orange squash and you mix one part orange to four parts water:
Ratios are usually written in the form a:b	the ratio of orange to water will be **1:4** (1 to 4)

Understand and Apply

If I want to make pink paint, I need to mix 3 cans of red paint with 2 cans of white paint. How will this be written as a ratio?

Crucial Knowledge
Dividing In A Given Ratio

Crucial Knowledge	Remember More
The simple method of dividing in a given ratio is to:	Share £400 in ratio 5:3
a) Add the ratios b) Divide the amount by the added ratios. c) Multiply each ratio part by step (b) d) Check your answers add to the original amount.	a) 5 + 3 = 8 b) 400 ÷ 8 = £50 c) 5 x £50 = **£250** 4 x £50 = **£200** d) £250 + £200 does make £450

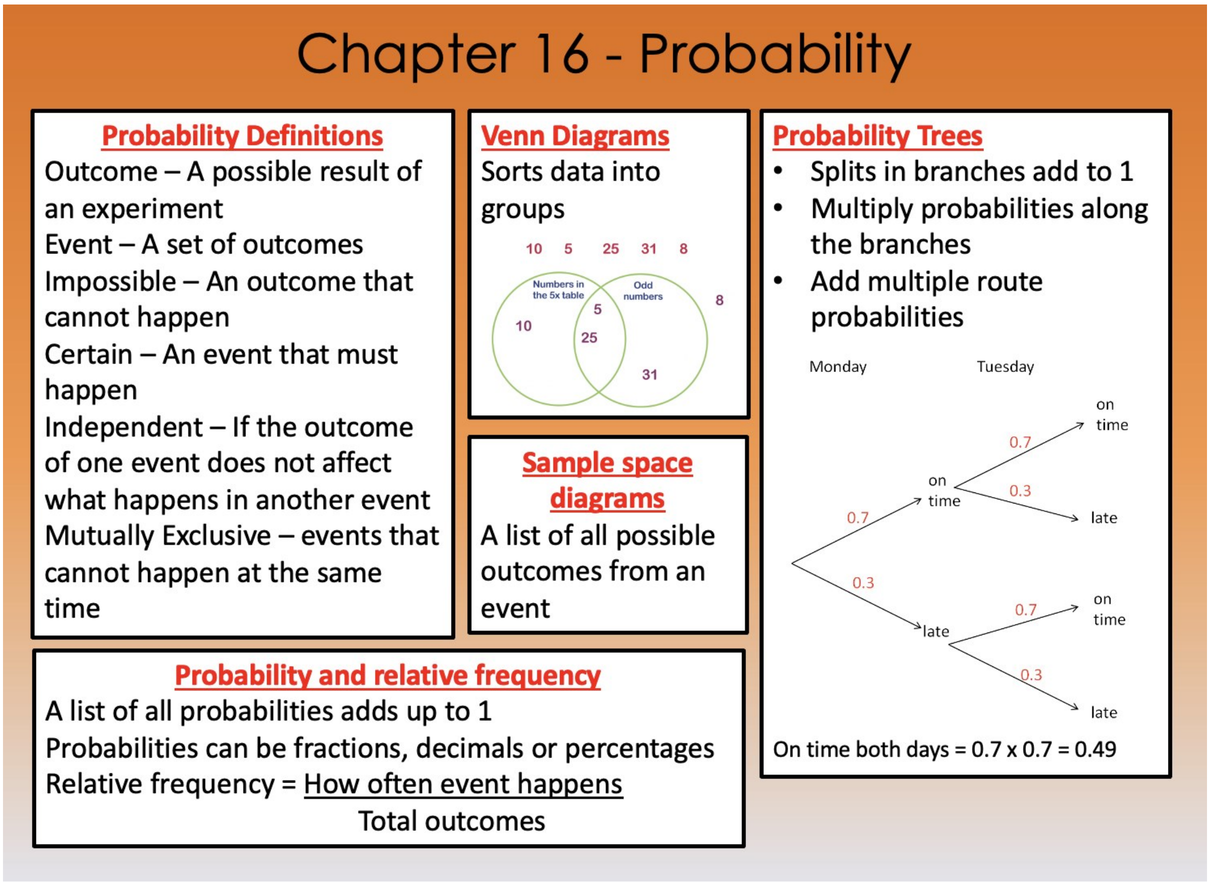
Understand and Apply

1) Share £450 in the ratio 3:2

2) Share £270 in the ratio 4:5

3) Share £560 in the ratio 5: 3

Probability



Chapter 16 - Probability

Probability Definitions

Outcome – A possible result of an experiment

Event – A set of outcomes

Impossible – An outcome that cannot happen

Certain – An event that must happen

Independent – If the outcome of one event does not affect what happens in another event

Mutually Exclusive – events that cannot happen at the same time

Venn Diagrams

Sorts data into groups

Sample space diagrams

A list of all possible outcomes from an event

Probability Trees

- Splits in branches add to 1
- Multiply probabilities along the branches
- Add multiple route probabilities

On time both days = 0.7 x 0.7 = 0.49

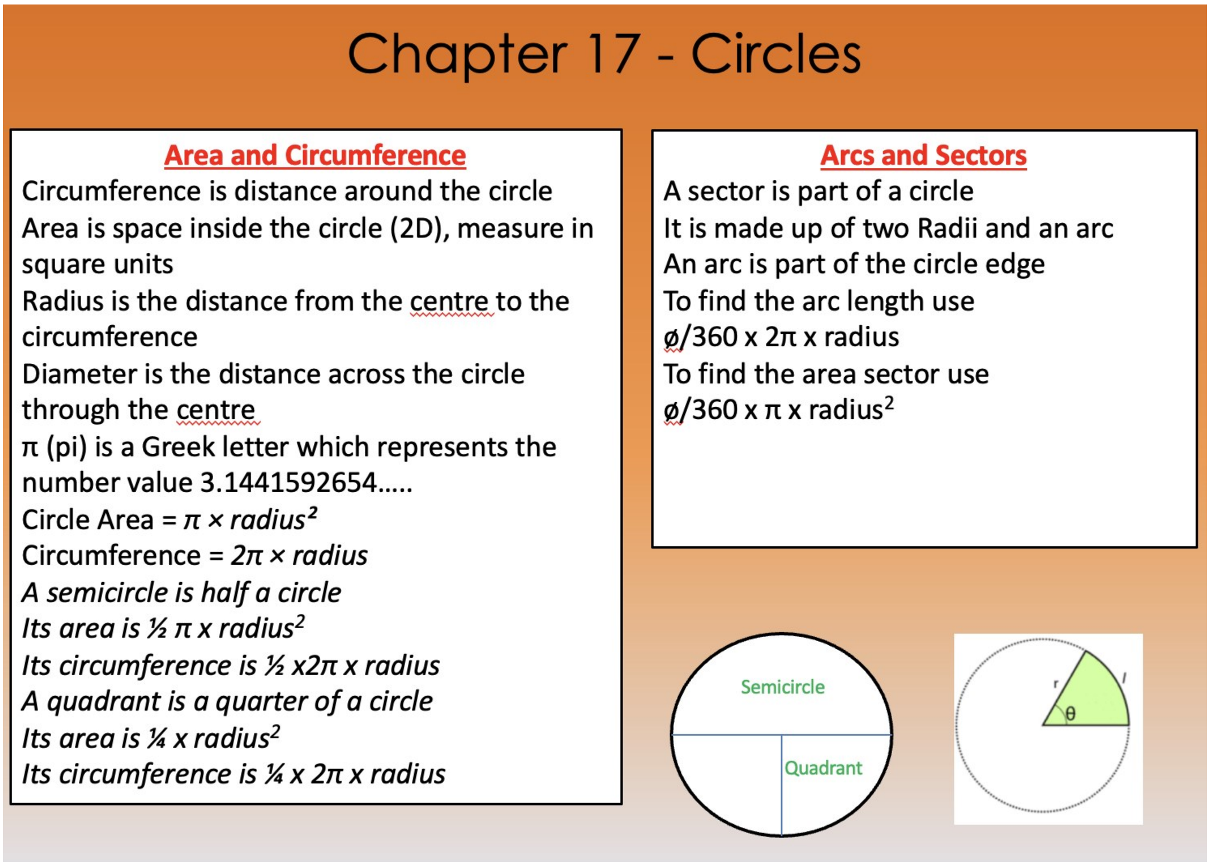
Probability and relative frequency

A list of all probabilities adds up to 1

Probabilities can be fractions, decimals or percentages

Relative frequency = $\frac{\text{How often event happens}}{\text{Total outcomes}}$

Circles



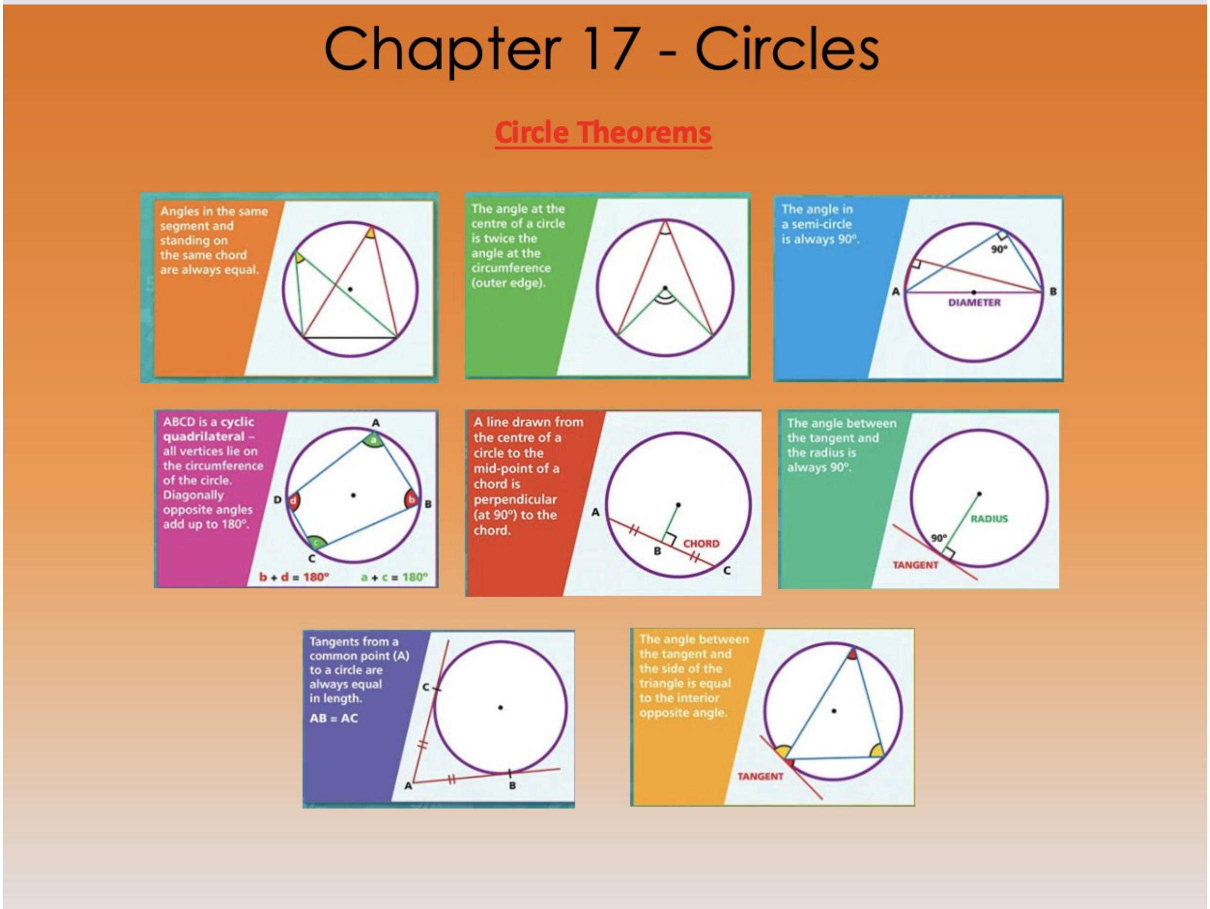
Chapter 17 - Circles

Area and Circumference

Circumference is distance around the circle
Area is space inside the circle (2D), measure in square units
Radius is the distance from the centre to the circumference
Diameter is the distance across the circle through the centre
π (pi) is a Greek letter which represents the number value 3.1441592654.....
Circle Area = $\pi \times radius^2$
Circumference = $2\pi \times radius$
A semicircle is half a circle
Its area is ½ π x $radius^2$
Its circumference is ½ x2π x radius
A quadrant is a quarter of a circle
Its area is ¼ x $radius^2$
Its circumference is ¼ x 2π x radius

Arcs and Sectors

A sector is part of a circle
It is made up of two Radii and an arc
An arc is part of the circle edge
To find the arc length use
ø/360 x 2π x radius
To find the area sector use
ø/360 x π x $radius^2$



Chapter 17 - Circles

Circle Theorems

Angles in the same segment and standing on the same chord are always equal.

The angle at the centre of a circle is twice the angle at the circumference (outer edge).

The angle in a semi-circle is always 90°.

ABCD is a **cyclic quadrilateral** – all vertices lie on the circumference of the circle. Diagonally opposite angles add up to 180°.

b + d = 180° a + c = 180°

A line drawn from the centre of a circle to the mid-point of a chord is perpendicular (at 90°) to the chord.

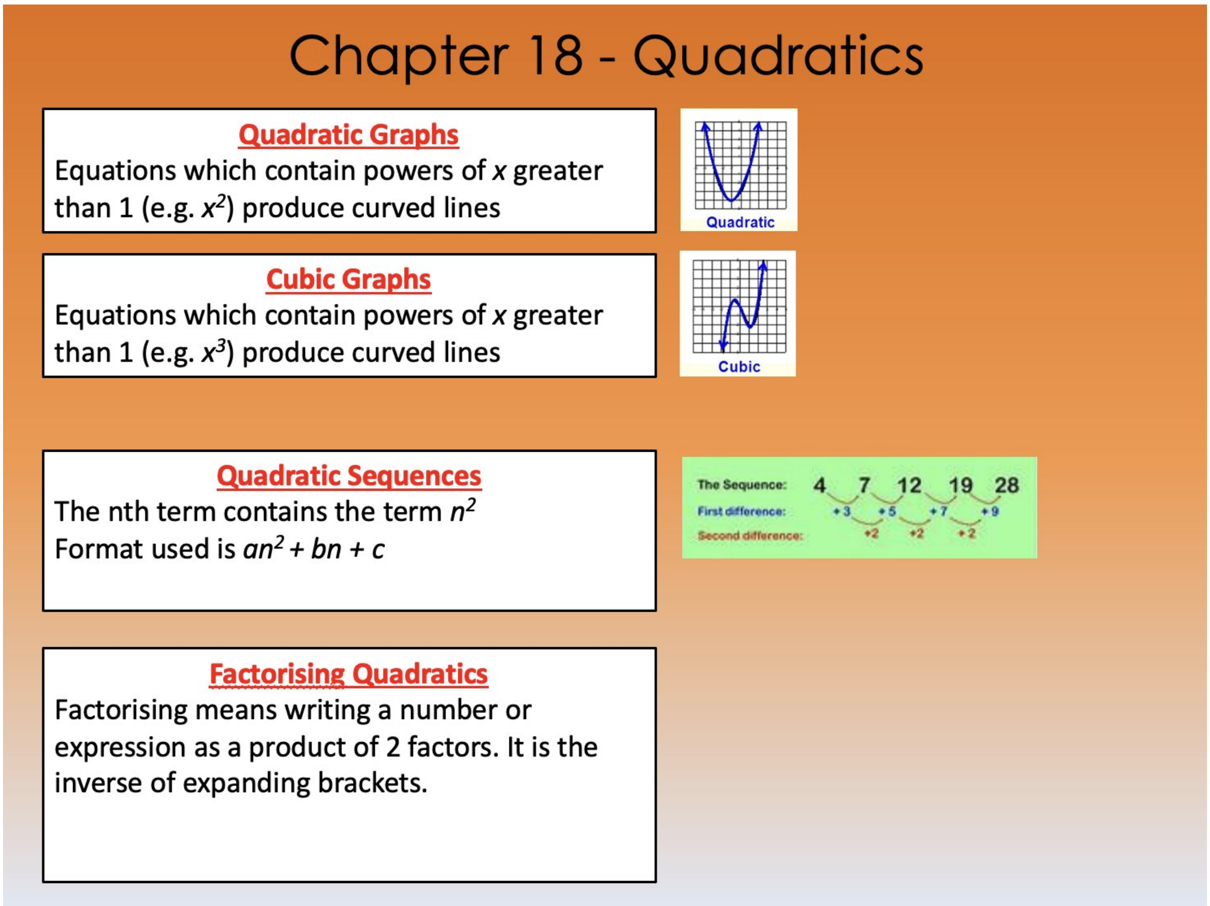
The angle between the tangent and the radius is always 90°.

Tangents from a common point (A) to a circle are always equal in length.

AB = AC

The angle between the tangent and the side of the triangle is equal to the interior opposite angle.

Quadratics



Chapter 18 - Quadratics

Quadratic Graphs

Equations which contain powers of *x* greater than 1 (e.g. x^2) produce curved lines

Cubic Graphs

Equations which contain powers of *x* greater than 1 (e.g. x^3) produce curved lines

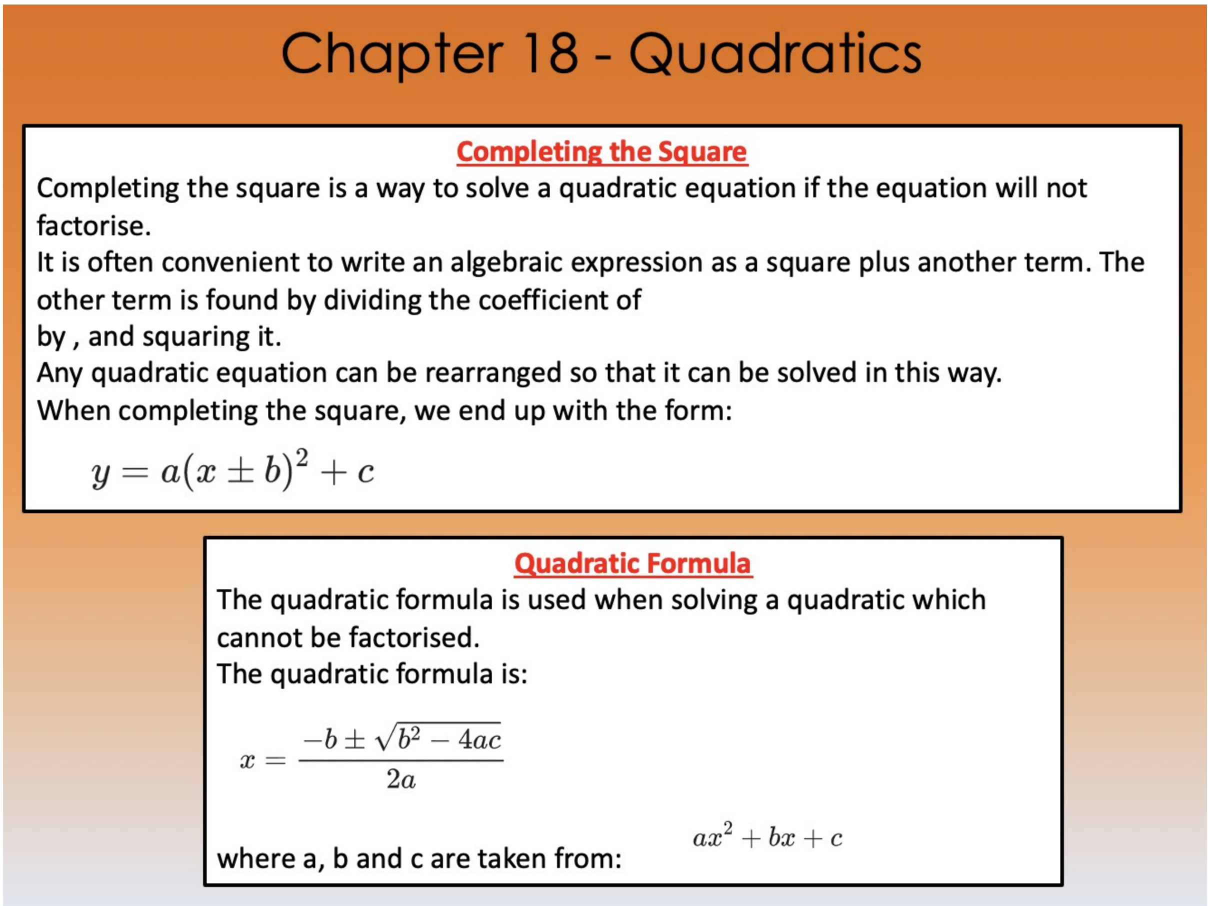
Quadratic Sequences

The nth term contains the term n^2

Format used is $an^2 + bn + c$

Factorising Quadratics

Factorising means writing a number or expression as a product of 2 factors. It is the inverse of expanding brackets.



Chapter 18 - Quadratics

Completing the Square

Completing the square is a way to solve a quadratic equation if the equation will not factorise.

It is often convenient to write an algebraic expression as a square plus another term. The other term is found by dividing the coefficient of

by , and squaring it.

Any quadratic equation can be rearranged so that it can be solved in this way.

When completing the square, we end up with the form:

$$y = a(x \pm b)^2 + c$$

Quadratic Formula

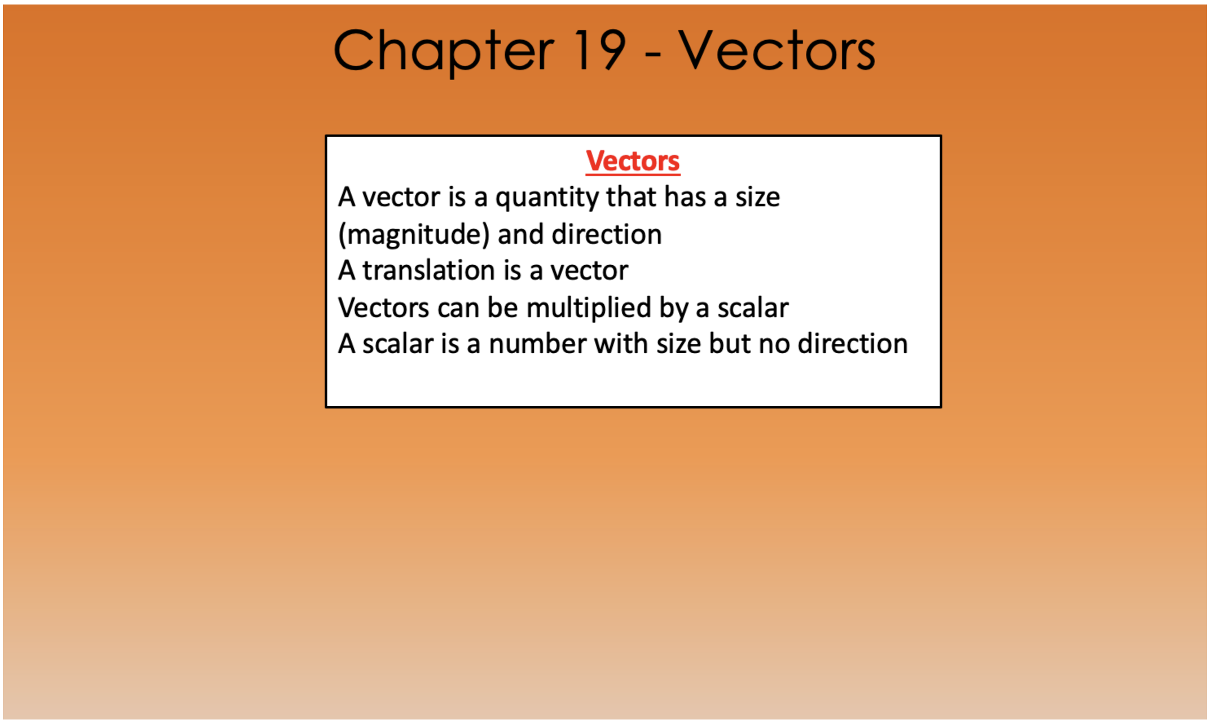
The quadratic formula is used when solving a quadratic which cannot be factorised.

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b and c are taken from: $ax^2 + bx + c$

Vectors



Chapter 19 - Vectors

Vectors

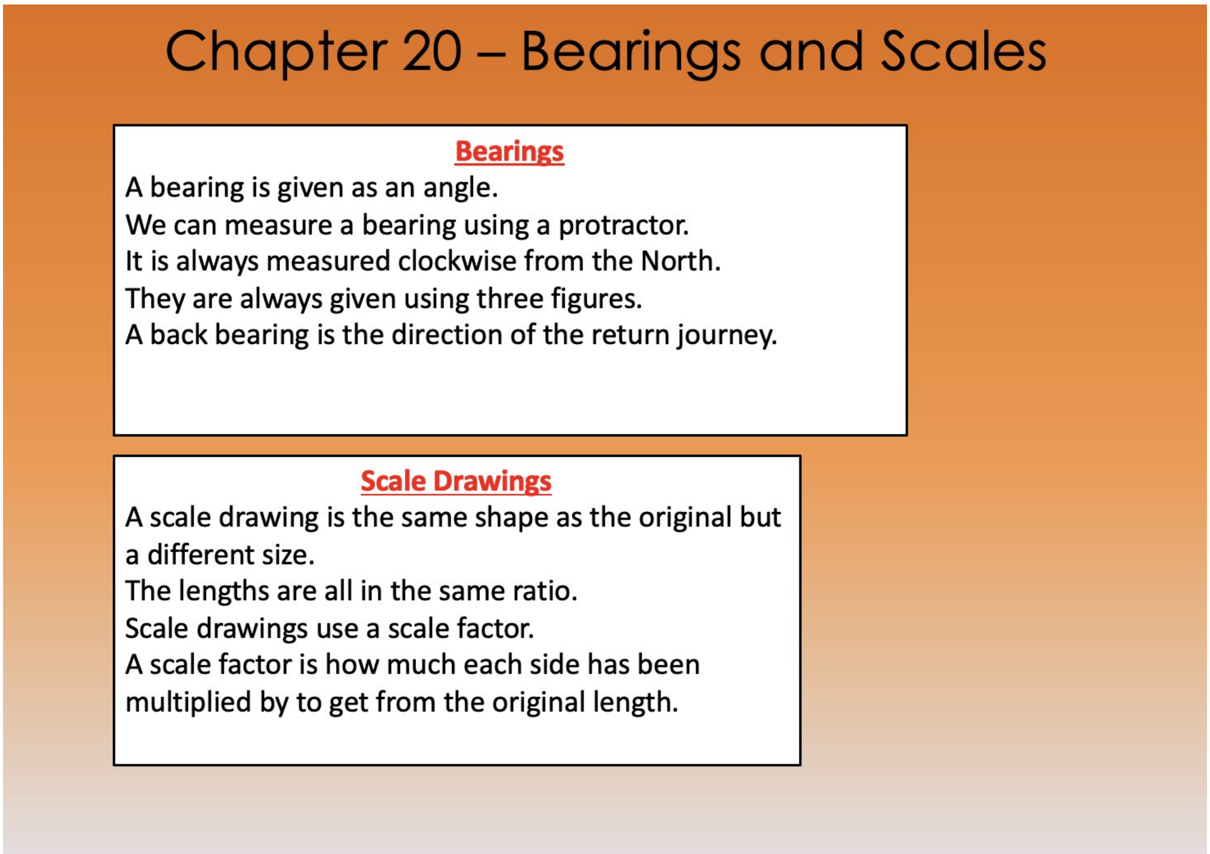
A vector is a quantity that has a size (magnitude) and direction

A translation is a vector

Vectors can be multiplied by a scalar

A scalar is a number with size but no direction

Bearings and Scale



Chapter 20 – Bearings and Scales

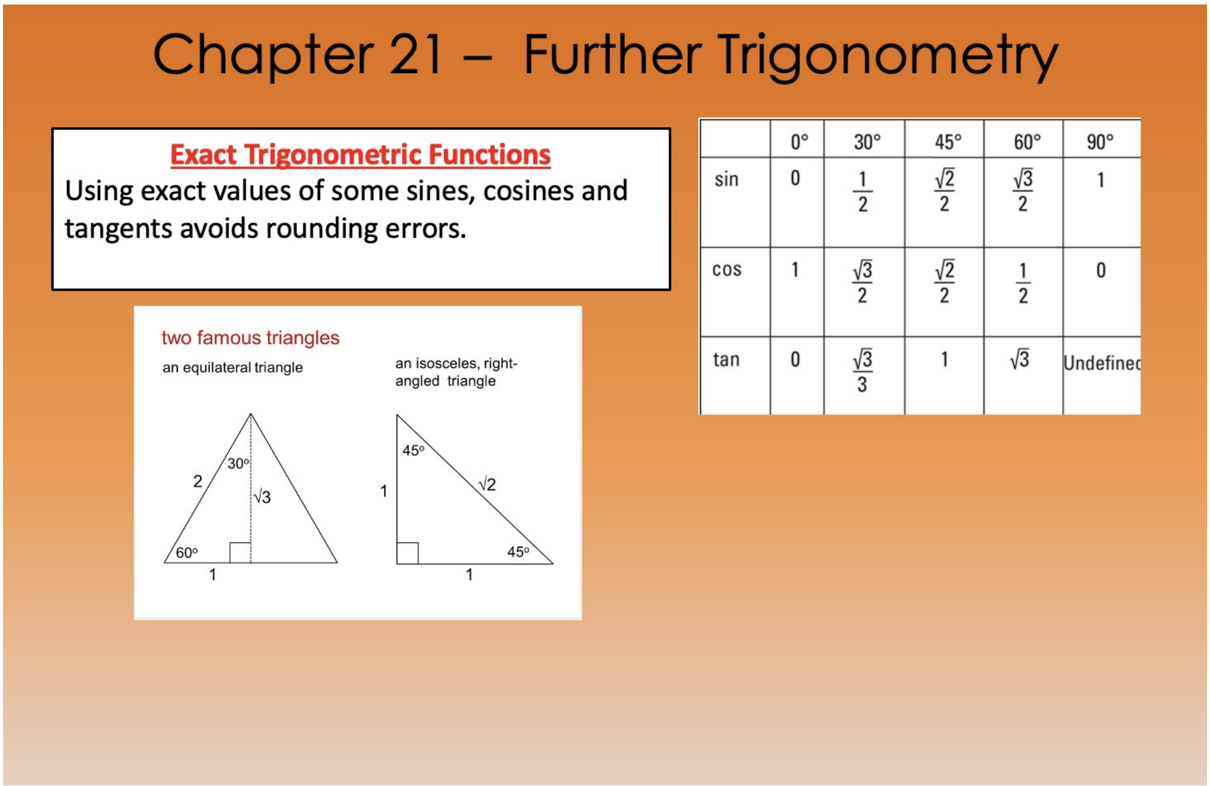
Bearings

A bearing is given as an angle.
We can measure a bearing using a protractor.
It is always measured clockwise from the North.
They are always given using three figures.
A back bearing is the direction of the return journey.

Scale Drawings

A scale drawing is the same shape as the original but a different size.
The lengths are all in the same ratio.
Scale drawings use a scale factor.
A scale factor is how much each side has been multiplied by to get from the original length.

Further Trigonometry



Chapter 21 – Further Trigonometry

Exact Trigonometric Functions

Using exact values of some sines, cosines and tangents avoids rounding errors.

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefinec

two famous triangles